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FILTRATION RESISTANCE OF SEVERAL COMMERCIAL FILTER AIDS

E. Robert Baumann, et al

Iowa State University
Ames, Iowa

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FINAL REPORT
PART 1 OF IV PARTS

FILTRATION RESISTANCE OF SEVERAL COMMERCIAL FILTER AIDS

E. Robert Baumann
Professor of Civil Engineering
Madan Lal Arora
Research Graduate Assistant
Charles S. Oulman
Associate Professor of Civil Engineering

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INTRODUCTION

General

Diatomite and perlite filter aids are commonly used as filter media in potable water filtration. Diatomite is the common name for diatomaceous earth. It is composed principally of silica (SiO_2) and consists of skeletal remains of microscopic, aquatic plants called diatoms. Alive, these diatoms are invisible as units but recognizable in colonies as gelatinous growths, either free floating or adhering to rocks and larger vegetation.

Deposits of diatomite are mined in various parts of the world including Canada and the western part of the United States. The Lompoc area in California contains the world's largest deposits of diatomite. Millions of years ago much of the present day California coast line was below sea level. The Lompoc area may have been an archipelago of low-lying islands or a series of lagoons or bays favorably located for the quiet deposition of sediments. These diatoms encountered a combination of conditions conducive to their rapid and continued production. Temperature, food supply and light conditions afforded the optimum encouragement to diatom growth. The waters were rich in silica due to the activity of submarine volcanoes. The diatoms extracted silica from the sea water, and thus, the diatom lived within a silicious cell wall. When the diatom life cycle was completed, their silicious cells remained as an inorganic mineral in the form of sediment at the bottom of the body of water.

Approximately 10,000 distinct species of diatoms varying in size from 5 to over 100 microns have been identified. After quarrying, the diatomite is processed for use as a filter aid by drying, crushing,

calcination, and separation into various grades on the basis of particle size distribution and filtering quality.

The basic physical properties of the diatoms must be considered first as they apply to the individual particle and, secondly, as they apply to a mass of particles. Diatoms are characterized by their irregular shapes, spiny structure, and pitted surfaces. See Figs. 13 and 14, Appendix I-D. These tiny individual particles, averaging only a few microns in diameter, are rigid and relatively strong. Manufacturers claim that this structure is preserved in milling. Loose diatom fossils in a mass do not pack together. Contact with each other is limited largely to the outer points of each particle involved. They are not compressible unless subjected to relatively great force[1].

Based on the true specific gravity of opaline silica (2.00-2.30, average of 2.15), a cubic foot of this material should weigh about 135 lbs. The apparent dry, bulk density, however, averages 10 to 20 lbs/cu ft. It is evident, therefore, that a large percentage of the apparent volume of diatomite must consist of a myriad of tiny interconnected pores or voids as shown in Figs. 13 and 14, Appendix I-D. This physical structure contributes a highly absorptive nature and a high specific surface to the material. A few typical Celite* properties are tabulated in Table 1[1].

Several commercial grades of diatomite filter aids are available from each of several different manufacturers. See Appendix I-D. These vary in particle size distribution, density, color and hydraulic characteristics. There may be a significant variation in these properties, between different shipments, even for the same grade of diatomite, due to the variations in the basic diatom structure within a deposit or

---

*Registered trademark, diatomite filter aids, Johns-Manville Products Corp., New York, N.Y.

Table 1. Typical Celite properties.

Physical properties		Chemical properties	
Specific gravity	2.0 -2.3	Average analysis in %, dry basis	
Refractive index	1.42-1.48		
Specific heat, cal/g°C	0.24	Silica (SiO_2)	89.7
Particle charge	negative	Alumina (Al_2O_3)	3.7
Retained on 325 mesh, maximum	0.5 -3.0%	Iron oxide (Fe_2O_3)	1.5
Average absorption, % Gardner-Coleman method		Titanium oxide (TiO_2)	0.1
		Lime (CaO)	0.4
Water	150-220	Magnesia (MgO)	0.7
Linseed oil	120-205	Alkalies (as Na_2O)	0.8
Bulk density, lb/cu ft		Ignition loss (combined H_2O, CO_2 and organics)	3.7
Dry, loose	8-10		
In filter cake	15-28		

from one deposit to another. There may even be variations within different lots of the same shipment due to variations in the processing conditions during a production run of a particular grade of material.

Perlite is a material outwardly similar in hydraulic characteristics to diatomite. Perlite is a silicious rock containing 3 to 5% water[2]. When crushed and heated during processing, the rock expands to provide a mass of glass bubbles, producing a lightweight aggregate with many sealed cavities which retard the conduction of heat and sound when used as loose fill or as aggregate in concrete and building plaster. Perlite

is also used as a filter aid, in horticultural applications, in oil-well cementing and grouting, and as an abrasive.

In commercial usage, the term "perlite" refers to any naturally occurring glass of igneous origin that expands when quickly heated to yield a frothy, light-colored mass of glass bubbles. See Fig. 15, Appendix I-D. The term is also applied to the expanded product[2].

The petrological definition of perlite is restricted to volcanic glass which has a concentric or "onion" structure of fracturing. Commercial crude perlite contains 3 to 5% water, has a waxy to pearly luster, is transparent to translucent at thin edges and is rhyolitic in composition. It is essentially an amorphous aluminum silicate of volcanic origin. An average analysis, in percent, of 10 perlites from 6 currently producing states is included in Table 2[2].

Several commercial grades of perlite filter aids are available from each of several different manufacturers. See Appendix I-D. These vary principally in their particle size distribution and in the specific gravities of their individual particles. As with diatomite filter aids, there may be significant variation in the filter aid characteristics of the same grade of material between shipments and even within the same production run due to variations in the ore and its processing.

Table 2. Typical perlite properties.

Physical properties[a]		Chemical properties[2], percent by weight	
Specific gravity	1.70-2.10	Moisture loss at 105°C	0.20
Particle charge	negative	Total moisture loss after ignition at 800°C	3.83
Bulk density in a filter cake, lb/cu ft	9.5-13.5	Aluminum oxide (Al_2O_3), including any phosphorous pentoxide or manganese oxide	13.08
		Lime (CaO)	0.72
		Iron oxide (Fe_2O_3)	0.89
		Magnesia (MgO)	0.18
		Potassium monoxide (K_2O)	4.44
		Silicon dioxide (SiO_2)	73.20
		Sodium monoxide (Na_2O)	3.31
		Sulphur trioxide (SO_3)	0.04
		Titanium dioxide (TiO_2)	0.09

[a]From the laboratory tests conducted at Iowa State University.

Background

The first practical use of diatomite for filtration of water was made during World War II to serve American troops in the Pacific area. The filtration equipment used by the US Army prior to World War II consisted of pressure sand filters which were not completely effective in removing Entamoeba histolytica cysts from water. These organisms cause amoebic dysentery and in cyst form are resistant to chlorination. This

health menace was recognized and a research program was undertaken to develop a better filtration system. Extensive research by the US Army Engineer Research and Development Laboratories (ERDL) demonstrated that diatomite filtration was effective in removal of these cysts and offered additional advantages in the area of logistics and versatility. The success of diatomite filtration in military applications stimulated its application to civilian uses in the late 1940's and 1950's, principally for the filtration of swimming pool waters. Since the design and operation techniques of diatomite filters were not fully developed, many such applications failed to provide an acceptable effluent at a cost comparable to sand filtration. However, with continued research and the development of better filter design and operating techniques, civilian use of such filters has increased steadily. A Task Group report on diatomite filtration[3] disclosed that since 1949, 85 diatomite filtration plants have been constructed in the United States. The majority of these plants is employed for the removal of turbidity from surface waters. As more experience with precoat filtration becomes available, such use is expected to increase.

No extensive research data on filtration using perlite filter aids is yet available. The manufacturers of perlite filter aids, however, claim that perlite and diatomite filter aids are comparable materials and that perlite filter aids will produce filtration results as good as those obtained using diatomite. They propose that perlite filter aids will take over a significant portion of the filter aid market now held by the diatomite industry. Data on the number of potable water filtration plants using perlite filter aids are not available.

REVIEW OF LITERATURE

General

Natural waters contain different types of suspended impurities such as clay minerals, iron and living organisms. These impurities are removed from suspension during passage of the water through a filter media by mechanical straining and by other phenomena. Particles larger than the openings in the filter cake are removed by a simple straining action, whereas the finer colloidal particles are removed by more complicated phenomena, including adsorption. It follows that more suspended material can be removed from water by using finer grades of filter aid which have smaller passages for the flow of water to enhance straining and a larger surface area to enhance adsorption. However, the filter runs which result will be shorter since there is a higher headloss across a filter using the finer grade of filter aid than using a coarser one.

The objective of providing a quality filtrate with economy is defeated if an engineer decides to use finer grades of filter media in preference to coarser ones, irrespective of the nature of the impurities in the water, the standard of effluent quality desired, and other pertinent factors relevant to any particular situation. The successful use of filters using filter media of the diatomite-perlite type depends on a knowledge of:

- The characteristics of the water to be filtered, including the characteristics of the solids to be removed,
- The characteristics of the filter media used to remove the solids effectively and economically, and
- The characteristics of the filtration system to be used.

Phillips[4] in 1957 summarized all of the work done in the field of precoat filtration to serve as a single consolidated review of the progress in this field to that date. Much additional work has been done on this process in recent years which makes the theory, design, and operation of such filters better understood than ever before.

A study by Sanchis and Merrell[5] in 195' revealed that precoat filters could be successful in solving taste and odor problems caused by algae growths in storage reservoirs. Diatomite filters produced water of quality comparable to that from conventional sand filtration plants, at no greater cost.

Baumann[6] studied all municipal filtration plants in the US using diatomite in 1957. He sent a questionnaire to health departments of all the states to determine their attitude, experiences, and opinions regarding municipal diatomite filtration plants. At that time, 6 states favored adoption of diatomite filters, 10 states reported that they would approve such installations with reservation, 20 states disapproved their use, and the rest stated that they had no fixed policy on the subject.

Much of the recent research involving filtration using diatomite and perlite filter aids has been conducted at Iowa State University. Studies have been made using pilot plants for the removal of iron[7-9], various flocculent solids[10-12], and clays[13]. Based on the results of these studies and on field experience, a theory of diatomite filtration has been developed.

Theory of Precoat Filtration

Before reviewing the development of the theory of precoat filtration, a few terms that appear in the discussion should be explained briefly:

1. Precoat filter: This term is used herein to describe the filter which uses either diatomite or perlite filter aids as the filter medium.

2. Precoat: Before starting a filtration cycle, a thin layer of filter aid is deposited on the outside surface of the septa located within a filter housing. The filter aid so deposited is used as the filter medium and is referred to as the precoat or as the precoat filter aid.

3. Septum: The cylindrical shell or other structure designed to serve as a supporting medium for the precoat.

4. Body feed: During a filter run, the pores at the surface of the precoat become clogged by the impurities in the influent water which are removed during passage of the water through the precoat. Such loss of permeability in the precoat is prevented by adding diatomite or perlite filter aid in suitable proportion to the suspended solids in the incoming raw water throughout the filter run. The filter aid applied in this way to the raw water is known as body feed. Without the addition of body feed, the filter runs would be very short and cracks forming in the precoat might cause a deterioration in the effluent quality.

5. Filter cake: The filter media collected on the septa, consisting of the precoat filter aid and the body feed and suspended solids removed by the precoat, is referred to as the filter cake.

6. Filter run or simply run: This term refers to the complete filtering cycle and consists of three distinct operations: a) precoating the septa, b) filtering the raw, turbid water and c) backwashing the septa. In the last operation, the filter housing is filled with clean water and then the water and any filter cake contained in it is allowed to run with high velocity to waste through a waste valve thereby cleaning the septa of the filter cake formed during operations a) and b).

The flow of water through a precoat filter occurs in the laminar flow range. Therefore, Darcy's law for laminar flow through a porous medium is applicable. Darcy's law states

$$V = KS \tag{1}$$

where

V = velocity of flow

K = coefficient of permeability

S = hydraulic gradient = $\frac{\text{headloss through filter cake}}{\text{filter cake thickness}}$.

The headloss through a filter cake consists of the headloss through two different types of cakes: headloss through the precoat and headloss through the cake deposited on the precoat composed of the body feed and the impurities in the raw water which are filtered out.

On such a basis, LaFrenz[7] developed an equation for the headloss through the filter cake during a filter run. This equation was later modified and published[14] in the following form:

$$H = K_3 Q W_1 + K_4 C_F Q^2 t (8.33 \times 10^6) \tag{2}$$

where

K_3 = a factor that depends upon the coefficient of permeability and the in-place bulk density of the precoat. K_3 has an inverse relationship with the coefficient of permeability and has units of $(min)(ft^5)/(lb)(gal)$

K_4 = a factor that depends upon the coefficient of permeability and the in-place bulk density of the body feed layer. K_4 also is inversely related to the coefficient of permeability and has the same units as K_3.

Q = filtration rate expressed in gal/min ft^2 of septa area

W_1 = weight of precoat in lb/ft^2 of septa area

C_F or C_D = concentration of body feed in mg/l or ppm

t = time in minutes elapsed since the start of the filtering cycle in a run.

8.33 lbs/gal and 10^6 are necessary conversion factors to provide dimensional correctness of the equation.

The first term in Eq. (2) represents the headloss through the precoat. The second term represents the headloss through the body feed layer. In deriving this equation, it is assumed that the effect of the suspended solids on the filter cake thickness is negligible. Since the concentration of suspended solids in the raw water is generally a small fraction of the body feed concentration, this simplifying assumption does not create serious errors. The headloss through the precoat equals the initial headloss observed at the beginning of a filter run. This remains constant during the run and is a function of the grade and type of filter aid used. The headloss through the layer of suspended solids and body feed increases during a filter run. It is zero at the beginning of

the filtering cycle and is maximum at the end of the filter run when the cake has built to its maximum thickness. The headloss (to 100-150 ft of water) through the layer of suspended solids and body feed is much more significant than the headloss (1 to 2 ft of water) through the precoat.

Baumann et al.[14] reported values of K_4 observed in several filter runs using clear water to which various amounts of ferrous sulphate were added. They reported that K_4 values plotted against the ratio of the suspended solids concentration to the body feed concentration (C_S/C_D) produce a straight line on log paper. This was expressed as

$$K_4 = A(C_S/C_D)^B \qquad (3)$$

where

A, B = constants

C_S = suspended solids concentration in the raw water, (mg/l or ppm).

The validity of this equation was verified for most practical applications in a number of studies at Iowa State University[10,11,13].

Hawley[11] found that K_4 values obtained using synthetic raw waters containing trivalent iron and aluminum salts were about 8 to 10 times higher than those obtained from the same water but containing the same concentration of iron but from bivalent iron salts. Hall[10] added ferrous sulphate to water and oxidized the iron by using compressed air to produce hydrous ferric oxide floc. This synthetic water was used to determine the resistance of filter cakes resulting from the filtration of the same water using 4 different grades of diatomite from one manufacturer. He found that an increase in the K_3 values of the filter aid used of 450% obtained by using finer grades of diatomite, resulted in only a 48% increase in the K_4 values at a C_S/C_F ratio of 0.09.

Regunathan[13] continued the work and used a synthetic water containing clay minerals. He found that the straight line relationship between log K_4 and log C_S/C_F held for the kaolinite but not for the montmorillonite clay used. The K_4 values obtained using montmorillonite clay as the suspended solids were about 10 times greater than the K_4 values obtained using kaolinite clay at equal ratios of C_S/C_F.

The values of K_3 and K_4 are not true constants but are temperature dependent. Temperature affects both the density and viscosity of water and, therefore, both K_3 and K_4. Baumann and Oulman[15] published a modified equation to correct for changes in viscosity. This equation, although an improvement over the earlier equation, still did not include a few important factors inherent in a practical precoat filtration process and hence needed further modification.

At the end of a precoating operation, the filter housing is usually full of clean water. During the filtration cycle the water to be filtered gradually replaces this clean water and some time is required before the clean water is entirely replaced. As a result, the headloss development during this period early in a filtration run, known as the initial dilution period, is much less than it is after the housing is completely full of the raw water to be filtered.

Furthermore, when filtering water through cylindrical septa, the increasing cake thickness increases the surface area of the filter cake and provides less cake thickness per unit of water filtered. This causes the flow rate per unit area of the cake to diminish as the run progresses and the surface area increases. The decrease in flow rate per unit area which thus results during the run causes the rate of headloss increase

to diminish continuously throughout the run. This effect is known as an "increasing area effect." It follows, therefore, that the increasing area effect will be more predominant using cylindrical septa with smaller diameter than using septa with larger diameter for the same concentration of suspended solids and body feed and at the same flow rate.

Dillingham et al.[16] modified the filtration equation to include the effects of viscosity, initial dilution, and the increasing area for cylindrical septa. The modified equations are summarized in Table 3.

The equations in Table 3 are valid for use with any consistent set of units for the dimensions of force, length and time. In deriving these equations, the following hypotheses were assumed to be true during a filter run[16]:

1. Q remains constant (constant-rate filtration).
2. The body feed rate is sufficient to form an incompressible filter cake.
3. The in-place bulk densities of the precoat (γ_p) and filter cake (γ_c) remain constant.
4. The concentration of solids being filtered (C_S) and the body feed concentration (C_D or C_F) remain constant.
5. No solids pass through the cake.
6. No concentration gradients exist in the filter housing (a completely-mixed system).
7. The suspended solids in the raw water retained in the filter cake do not increase the cake thickness appreciably.

Assumptions 1 through 4 are reasonably valid during a filter run. Assumption 5 should also be true since the filter cake in a successful

Table 3. Summary of precoat filtration equations.

	Equation	Equation number
Any septum	$H_p = q\nu\xi w/g$	(4)
Cylindrical septum	$H_c = (R_S\sigma/2)\ln(1 + R_S\,\phi\,x/R_o^2)$	(5)
	$L = \sqrt{R_o^2 + R_S\,\phi\,x} - R_S$	(6)
Flat septum[a]	$H_c = \sigma x$	(7)
	$L = L_p + \frac{\sigma}{2} x$	(8)
where:	$\delta = Q/V_f$	
	$\sigma = q^2\nu\beta C_D/g$	
	$R_o = R_S + L_p$	
	$\phi = 2q\gamma_w r_j (10)^{-6}/\gamma_p$	
	$L_p = w/\gamma_p$	
	$x = t - (1 - e^{-\delta t})/\delta$	

Symbol	Meaning	Dimension
A_s	Septum area	$[L^2]$
β	Filter cake resistance index or β-index	$[L^{-2}]$
C_D	Body feed concentration ppm by weight	$[-]$[b]
C_S	Suspended solids concentration ppm by weight	$[-]$[b]
j[c]	Dilution rate, theoretically Q/V_f	$[T^{-1}]$

Symbol	Meaning	Dimension
g	Gravity constant	$[LT^{-2}]$
γ_p	Bulk density of precoat	$[FL^{-3}]$
γ_c	Bulk density of filter cake	$[FL^{-3}]$
γ_w	Density of water	$[FL^{-3}]$
H_p	Head loss through precoat	$[L]$
H_c	Head loss through filter cake	$[L]$
L_p	Thickness of precoat	$[L]$
L_c	Thickness of filter cake	$[L]$
L	$L_p + L_c$	$[L]$
ν	Kinematic viscosity of influent	$[L^2T^{-1}]$
Q	Flow rate	$[L^3T^{-1}]$
q	Flow rate per unit septum area	$[LT^{-1}]$
V_f	Volume of housing	$[L^3]$
R_s	Outer radius of septum	$[L]$
R_o	Outer radius of precoated septum = $R_S + L_p$	$[L]$
t	Time since start of filtering cycle of a particular run	$[T]$
w	Precoat weight per unit area	$[FL^{-2}]$
x	$t - (1 - e^{-\delta t})/\delta$	$[T]$
ξ	Filter aid resistance index or ξ index	$[F^{-1}L]$

[a] Septum that does not exhibit increasing area effect.

[b] Dimensionless.

[c] δ will be equal to Q/V_f in a completely-mixed system in a filter housing. Since the conditions in the experiment are far from ideal, values of δ should be estimated by visual selection of the inflection points on the head loss versus time curve in any particular filter run with the assumption that at this time δt is approximately equal to 3[17].

filter run should not allow a significant fraction of the suspended solids to pass through. Assumption 6 is not always valid in a practical situation. In a normal filter the raw water entering a filter housing will be mixed rather well with the water within the housing. In such cases, conditions approaching complete-mix will prevail. In many cases, however, the degree of mixing will not be such as to provide a

uniform mixture in the housing and assumption 6 will be less valid. However, if the values of dilution factor, δ, are read from the headloss versus time curve as explained in Table 3, the assumption may be considered valid without affecting the results seriously. Since the concentration of suspended solids (C_S) is generally far less than the concentration of body feed (C_F), the last assumption is also true.

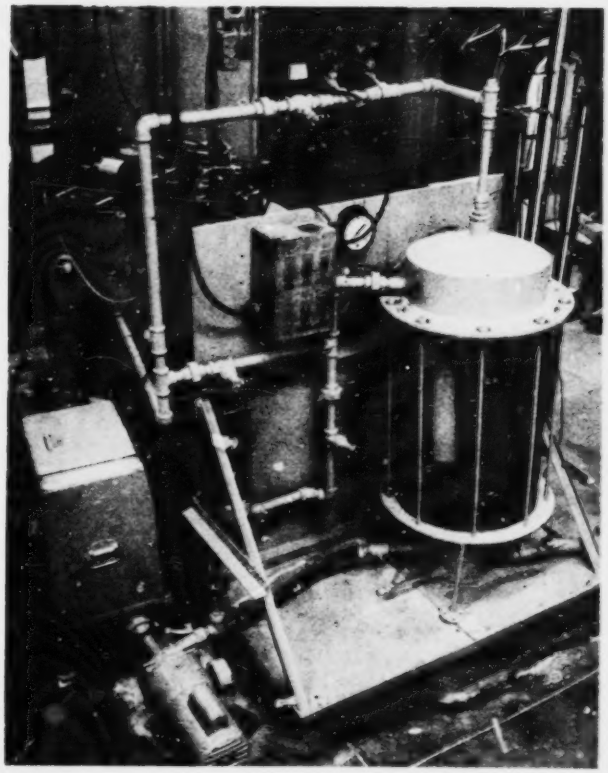

Fig. 1. The pilot precoat filtration plant.

The β-index in the equations in Table 3 measures filter cake resistance and depends on the nature of the suspended solids, their concentration (C_S) in the raw water, and the filter aid characteristics and concentration of body feed (C_F). Since all of these are constant during a filter run, the β-index will remain constant during the run. However, this index will vary from one run to the next as the raw water quality changes or as changes are made in C_F. All of the data required for computation of the β-index may be collected in a filter run with the exception of values of the bulk density of the precoat, γ_p, which can be determined separately (Appendix I-A)*.

*A separate description of a "Standard Procedure for the Determination of Bulk Density and Filter Cake Resistance of Precoat Filter Aids" is included as Part V of the Final Report. Values of γ_F determined using the standard procedure may be used in the determination of values of β from a filter run.

From a practical viewpoint, the engineer is interested in predicting filter cake resistance under such varying conditions as different values of C_S and C_F when filtering a particular water using a particular filter aid. To facilitate making such a prediction, Dillingham et al.[18] developed several β-index prediction equations which are summarized below:

$$\beta = 10^{b_1}(C_S/C_F)^{b_2} \cdot (C_F)^{b_3} \cdot \xi^{b_4} \tag{9}$$

$$\beta = 10^{b_1}(C_S/C_F)^{b_2} \cdot (C_F)^{b_3} \tag{10}$$

$$\beta = 10^{b_1}(C_S/C_F)^{b_2} \tag{11}$$

where

b_1, b_2, b_3, and b_4 are exponents determined empirically.

Use of the additional variable C_F in Eq. (10) as compared to Eq. (11) improved cake resistance prediction[17] in the case of some suspended solids. Either Eq. (10) or (11) can represent the results of a group of filter runs using the same grade of filter aid ($b_4 = 0$). In this study, β-prediction equations were generally used in the form of Eq. (11).

PURPOSE AND SCOPE OF THIS STUDY

The β-index, as a measure of filter cake resistance, is an important criteria needed in the design of an economical filtration plant. The β-index, as is evident from Eq. (9), depends on such filter aid characteristics as the porosity, grain size distribution, arrangement of pores, and bulk density of the filter aid in the cake since all these factors are accounted for in the term ξ. The β-index, when using a single grade of filter aid, is also a function of the filter cake composition, which depends upon the ratio of suspended solids concentration to the body feed concentration (C_S/C_F) and the nature of the suspended impurities.

There are many commercial filter aids (diatomite and perlite) available in today's market. The most desirable filter aid to use in a given filtration plant is that grade and type which provides the desired quality of filtered water at least cost.

Baumann et al.[19] observed that filtration cost is a function of a number of variables, the most important of them being the cost of filter aid, labor, power, and the fixed cost of the plant itself. The problem of selecting one filter aid in preference to any other is, in fact, a problem of individual optimization that must be tackled in each particular case after consideration of all the pertinent factors. There cannot be a solution with universal applicability. Each water and each locality presents its own problems.

A special computer program, a "Program for Optimization of Plant Operation," referred to as POPO, has been developed at Iowa State University to optimize precoat filtration plant design. POPO is used to determine the

optimum combination of unit flow rate, body feed concentration, and terminal headloss that will result in the production of filtered water at minimum cost.

Since this study was supported by the US Army Engineer Research and Development Laboratories, the comparative performance of filter aids would be most significant if their performance was related to the filtration of a water whose characteristics were typical of the effluent from military water supply pretreatment units such as the Erdlator. Such effluents contain a hydrated ferric oxide floc which is removed by the military precoat filters.

The purpose of this study, therefore, was to determine and compare the hydraulic characteristics of several commercial diatomite and perlite filter aids, under conditions simulating military-type water filtration operations so that the results could be used to determine a filter aid specification for military purposes.

The two important essentials of any filter aid in a military operation are that it produces water with acceptable quality and that it does so with the production of the greatest possible volume of water for each unit volume of filter aid used. This requires the use of a filter aid which provides a reasonable rate of headloss development. Over 300 filter runs were made using many different types and grades of filter aid. These runs, which simulated the operation of a military-type precoat filter, provided data for evaluating the effect of the filter aid on water quality and on filter cake resistance. The filtered water quality was measured by determining the residual suspended solids concentration in the filtered water. Similarly, the filter cake resistance was determined by using a

special computer program (BID program)* to calculate the β-index of cake resistance from the results of each filter run. Manual calculations of the β-index from the results of a filter run are included in Appendix I-B.

The filter aid which produces an acceptable quality of water and develops a minimum value of filter cake resistance, β, is the most desirable for use providing its cost at the site of use is not higher than that of other satisfactory grades. Actually, the filter aid which develops the least filter cake resistance and yet produces acceptable water may cost more per pound than another filter aid producing slightly higher filter cake resistance. The decision to select one particular filter aid over another may then be based on a cost comparison. The filter aid which costs the least to develop any desired filter cake resistance (βC_F) is the best providing it produces water of acceptable quality. However, the cheapest cost of water filtration, as pointed out earlier, results with an optimum combination of the type and grade of filter aid, unit flow rate, body feed concentration, and terminal headloss. This optimum combination differs for the different filter aids.

The purpose of this study was to provide data on which the optimum combination of the above factors can be calculated in a practical situation based on costs of power, labor, the filter aid, and the plant. In situations where some of the factors listed above cannot be changed (as is the case with existing military water treatment plants and operating procedures) the results of this study can be used to determine how much

*A "User Manual for the BID Program" is included as Part V of the Final Report.

of each type of filter aid is required to provide the same "equivalent performance." Equivalent performance is described in more detail later in this report.

To avoid erroneous conclusions which might result from experimental errors in the laboratory and from variations in the quality control during the processing of the filter aids by their producers, a minimum of three samples of each grade and type of filter aid were tested in the laboratory.

The specific steps followed to achieve the objectives of this study were:

1. A group of filtration runs was made in a simulated military-type operation using each type and grade of filter aid so that the only variable between runs in the same group was the body feed concentration (C_F). The variable between runs in different groups was the use of a different grade or type of filter aid.
2. The suspended solids concentrations in the raw and filtered water were determined in each run.
3. The value of the β-index was calculated from the data collected in each run. A special computer program, the "BID Program," was developed and used to compute this index of filter cake resistance.
4. Beta-prediction equations were developed for filtration of the same suspended solid using different types and grades of the filter aids. The β-prediction equations were determined from a

computer analysis of the β-indices determined in a group of runs using the same filter aid. A computer program called "MAIDS"* was developed and used for this purpose.

5. The performance of a filter aid was then compared on the basis of the filtered water quality and the β-index observed with each.

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*A "User Manual for Maids Program" is included as Part VI of the Final Report.

LABORATORY APPARATUS AND PROCEDURES

General

To evaluate the performance of filter aids in simulated military-type operations, a pilot-plant precoat filter was used to filter University tap water to which ferric chloride had been added. Standard army water treatment involves the pretreatment of raw water in an Erdlator before filtration. An Erdlator is an upflow, solids-contact water treatment device. In the pretreatment, ferric chloride is used as the flocculating chemical and limestone is added both to buffer the system and to weight the floc[20]. The treated water carries over residual floc in the form of a hydrated ferric oxide which is removed by the military filters.

The pilot plant, therefore, consisted of apparatus used to prepare a standard synthetic water containing hydrated ferric oxide and a precoat filter designed with all the appurtenances needed to collect data accurately for evaluation of the hydraulic resistance of the filter cakes and the qualities of the raw and filtered water. The qualities of the raw and filtered water were determined by making a chemical analysis of the concentration of the iron in the samples. The hydraulic resistance of the different filter cakes was determined by using the pilot plant to filter a standard synthetic raw water under standard pilot-plant operating conditions as follows:

1. Water temperature = 60° F,
2. Filtration rate = 1 gallon per minute per ft^2 of septa area,

3. Precoat weight = 0.15 lb per ft^2 of septa, and

4. Suspended solids concentration, C_S = 8.0 ($\pm$ 0.5) mg/l.

A group of filter runs was made using a given type, grade, and production lot of filter aid to determine the hydraulic characteristics of that material. In this group of runs, the pilot plant was operated under identical conditions except for the variation in the body feed concentration, C_F. The results of a single run were used to determine the β-index for the C_S/C_F conditions of that run. The β-index results from all of the runs in the group were used to determine the β-prediction equations for that filter aid under the standard test conditions.

Other groups of runs were made to determine the β-prediction equations for other filter aid types, grades, and production lots. The comparison of the performance characteristics of filter aids was made by evaluation of the variation in the β-index of each filter aid under equivalent (C_S/C_F) operating conditions.

Pilot Plant

The pilot plant used in this study has been operated for several years at Iowa State University for research in precoat filtration (Figs. 1 and 2). The pilot plant construction and operation have been described in detail by LaFrenz[7] and Miley[8] and will only be described briefly here. The pilot plant consists of a mix tank used for preparing the standard synthetic water, a precoating and body feeding system, a diatomite filter with appurtenances required to regulate and observe the flow rate and to measure the headloss across the filter cake.

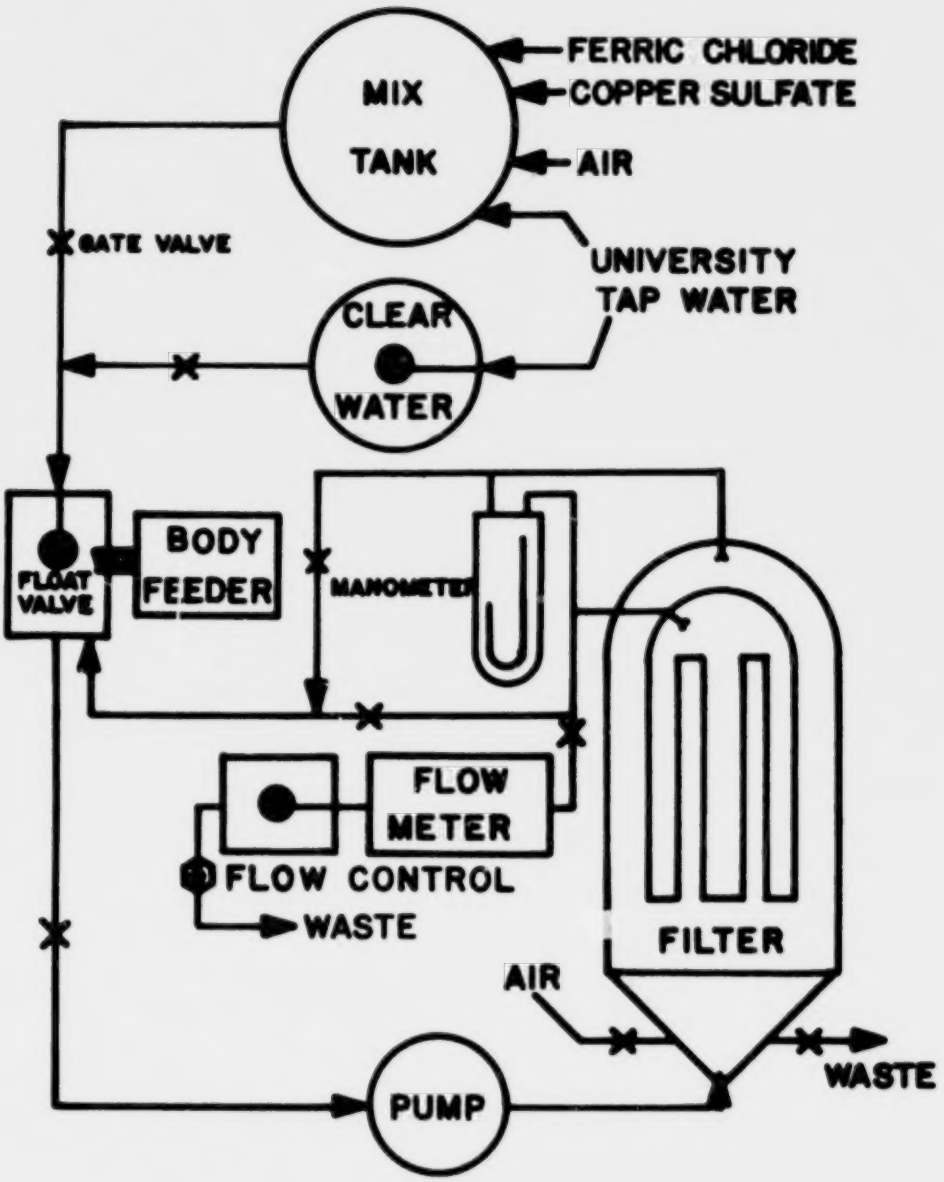

Fig. 2. Schematic diagram of the pilot plant.

The standard synthetic water prepared in the mix tank flows into a small body-feed mixing hopper through a float-controlled valve. The

body feed was added to the hopper through a screw-type volumetric feeder.* Two feed screws were provided with this equipment and either could be used depending on the body feed concentration desired. A time clock controlled the feeder to run for a predetermined portion of each minute. By using the proper screw and time setting, the body feed rate could be varied over the range studied, i.e. from about 50 mg/l to 500 mg/l. The water was then pumped from the hopper through the filters at a constant rate of 1 gpm/ft^2 of septa area. This flow rate was controlled using a flow regulator placed in the line after the filters. The flow rate was measured using a calibrated rotameter.

The filter had 3 cylindrical septa, each 3-1/2 in. in diameter and 14 in. long (total septa area of 3.2 ft^2). The septa were constructed with a synthetic fabricated sock stretched over a plastic frame. The septa are contained in a plastic filter housing which permits the viewing of the filter cake. The headloss across the filter cake and septa only was measured using a standard 100 in. mercury manometer.

Two recirculation lines are provided in the pilot plant. One of these lines connects the effluent side of the filter housing back to the hopper where the body feed is added and was used during the precoating operation only. The other recirculation line connects the interior of the filter housing on the unfiltered water side of the septa to the body feed hopper and was used both during precoating operation and during the filter run to provide an upward velocity in the housing high enough to facilitate formation of an even filter cake over the entire filtering area. During the precoating operation, clear water was drawn from a 32 gal tank which was connected through a float valve to the University water supply system.

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*BIF Model 25-01, B.I.F. Industries, division of New York Air Brake Company, Providence, Rhode Island.

Standard Synthetic Water

The standard synthetic water filtered in all runs was prepared in the mix tank (Fig. 2) by adding analytical grade ferric chloride, $FeCl_3 \cdot 6H_2O$, to University tap water. The mix tank was 36 in. in diam and 32 in. in height. Earlier studies indicated that a small amount of copper in solution resulted in a more complete precipitation of soluble iron[21]. Therefore, copper sulfate was added to the mix tank to provide a copper concentration of 1/2 mg/l. The standard iron concentration in the water to be filtered was 8 mg/l (range 7.5 to 8.5).

Operating Procedure

In making a filter run, a standard operating procedure was followed. First, the standard synthetic water was prepared on a continuous basis and brought to the standard conditions. A stock solution of ferric chloride was prepared by adding 2500 g of $FeCl_3 \cdot 6H_2O$ and 350 ml of concentrated sulfuric acid to 18 liters of distilled water. The purpose of the sulfuric acid was to dissolve the ferric chloride completely in the distilled water. This concentrated iron solution was then fed to the mix tank through a capillary tube at a drip rate calculated to yield 8 mg/l of iron (Fe^{+++}) in the synthetic water. Similarly, a copper sulfate stock solution was prepared by dissolving 180 g of $CuSO_4 \cdot 5H_2O$ in 18 liters of distilled water. The stock copper solution was also fed through a capillary tube at a drip rate designed to yield 1/2 mg/l of

copper in the synthetic water. Tap water* was added to the mix tank at a constant rate of 7 gpm using a Dole flow control valve to regulate the flow. The temperature of the water was maintained at 60° F. A thermostat** was placed in the mix tank to actuate a relay which in turn opened and closed a solenoid valve on the hot water line when the temperature dropped below the desired 60° F level. Since the filtration rate was only 1 gpm/ft^2 of septa area (i.e., 3.2 gpm for the 3.2 ft^2 of septa area provided in the pilot plant), the excess water in the mix tank overflowed to waste through a 2 in. diam overflow pipe.

The solution in the mix tank was mixed mechanically by a rotating paddle with a peripheral velocity of 2.5 ft/sec. An air diffuser was placed in the bottom of the mix tank to provide for supplemental aeration of the standard synthetic water, to insure complete mixing and to precipitate the iron completely to a hydrated ferric oxide floc.

The body feeder was emptied at the end of each run and refilled with fresh filter aid, since the results of a previous study indicated that the continuous motion of the screw in the body feeder degraded the filter aid particles slightly. By replacing the filter aid in the body feeder after each run, the degradation effects on the filtration

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*A typical mineral analysis of University tap water was reported by Regunathan[22]:

K+	2.6 ppm	NO_3-	4.3 ppm
Na+	18 ppm	F-	0.5 ppm
Ca++	105 ppm	Cl-	20.5 ppm
Mg++	36.4 ppm	SO_4--	110 ppm
Mn++	0.05 ppm	HCO_3-	364 ppm
Dissolved solids	528 ppm	Hardness as $CaCO_3$	412 ppm
Total solids	528 ppm	CO_3--	none
pH	7.75		

**Thermoswitch - thermostat, Fenwal Inc., Ashland, Mass.

results were minimized. To determine the body feed rate in each run, 10 min samples of body feed filter aid were collected as the filter aid left the body feeder both before and after each run. The average of the two rates of discharge was recorded as the run body feed concentration.

Once the synthetic water was being produced on a continuous basis under the standard conditions and the rate of body feed had been determined, the filter was washed, precoated, and a filter run started. The sequence of these operations was:

1. To wash the septa, clean tap water was drawn from the 32-gal clear water tank. A pump was used to fill the filter housing with the clean water which was then drained to waste.
2. To deposit the precoat on the septa, a weighed amount (0.15 lb of filter aid per ft^2 of septa area was used in all the runs) of filter aid was poured in the body feed hopper and pumped to the filter housing with both recirculation lines open. Clean water was used during the precoating operations. Recirculation was continued until a uniform layer of precoat was deposited on the septa and the water in the hopper contained no more filter aid.
3. To start the filtering cycle, the filter discharge was directed through the flow meter and flow control system. Simultaneously, the recirculation line that connected the effluent side of the housing to the hopper was closed. The discharge outlet of the body feed mechanism was brought over the body feed hopper

immediately. The flow rate of synthetic water through the filter was adjusted to 1 gpm/ft^2 of septa area. During the filtration cycle, the following data were observed and recorded at appropriate intervals:

- Headloss,
- Clock time, and
- Total iron concentration in both influent and effluent.

4. To backwash the filter at the end of a run, clean water was again used from the 32 gal clear water tank. Air was allowed to enter the filter housing at the bottom, as the filter housing drained, to break loose the filter cake which then drained to waste.

Analytical Techniques

A synthetic water containing precipitated hydrous ferric oxide floc was filtered in all filter runs. The qualities of the influent and effluent water were determined on the basis of their iron content. The influent samples were collected from the sampling tap provided in the bottom of the mix tank. The effluent samples were collected from the discharge end of the filtered-water effluent line. During each run at least 3 to 4 samples of influent and effluent were taken and analyzed.

The colorimetric-bipyridine method was used to analyze for iron. This is one of the standard methods adopted in the water supply field, is fairly sensitive, and the analysis can be performed rapidly[23]. A

bipyridine reagent in a patented powder form* dissolves the iron in water instantaneously at ordinary room temperature, producing a pink color which obeys Beer's Law. A spectrophotometer** was used to determine the percent transmittance of light passing through the solution. The wave length best suited for the purpose was 512.5 millimicrons. From the percent transmittance of light determined, the iron concentration in the water sample in mg/l could be determined using a laboratory prepared calibration curve.

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*Biver, Hach Chemical Co., Ames, Iowa.

**Model B. spectrophotometer, Beckman Instrument, Inc., Fullerton, Calif.

PRESENTATION AND ANALYSIS OF TEST RESULTS

General

Since this study involved a large number of filtration runs using many filter aids, it was necessary to design a suitable identification system for the filter runs and for the filter aids. The runs were numbered chronologically starting with number 1. The filter aid identification system developed and used is listed in Table 4. The letter

Table 4. Identification system for the filter aids.

Manufacturer	Grade of filter aid (coarse to fine)	Identification system designation
Eagle-Picher Industries, Inc.	FW-60	E6-xx
	FW-50	E5-xx
	FW-20	E2-xx
Great Lakes Carbon Corp. (Dicalite)	4200	G4-xx
	Speedex	G1-xx
Johns-Manville Products Corp.	Celite-545	J4-xx
	Celite-535	J3-xx
	Celite-503	J0-xx
Sil-Flo Corp.	272[a]	S2-xx
	332[a]	S3-xx
	443[a]	S4-xx

Example: A sample identified with designation J3-12 would be a sample taken from bag 12 of a Celite-535 shipment.

[a]Indicates perlite filter aids. Other filter aids listed in the table are diatomite.

and first number in the identification system designation identify the manufacturer and the grade of filter aid respectively. The second number preceded by a dash represents a specific bag or sample of that filter aid. Although not included here, this number is also correlated with the source of the shipment, the date received, manufacturer lot number, etc.

Determination of In-Place Bulk Density of the Precoat (γ_p) and the Dilution Factor (δ) of the Filter System

All of the data required for the computation of the filter cake resistance, or β-index of a filter cake, are collected in the actual filter runs except the values of the in-place bulk density (γ_p) for the filter aids used and the dilution factor (δ) for the filter system. Values of δ should be estimated by visual selection of the inflection point on the headloss versus time of filtration curve in any particular run. Using that method, the value of δ was calculated and found to average 18/hour in all the runs made in this study. This corresponds to the inflection point on the headloss versus filtration time curve at a filtration time of 10 min. So that values of β do not depend on the whims involved in drawing headloss versus time curves and in locating inflection points, a constant value of 18/hour for δ was used in analyzing the data from all of the runs. Since the value of dilution factor is not critical in the determination of β, an assumption of a constant value for this factor in all the runs will not significantly affect the results.

The in-place bulk densities of the filter aids used in this study were determined using the technique described in Appendix I-A. The in-place bulk densities of the filter aids used in calculating filter cake resistances are listed in Table 5. The in-place bulk densities used in the calculation of β-indices represent an average of from two to four determinations of the bulk density. The values reported in Table 5 may vary slightly from those reported in Part II of the Final Report in which the "Proposed Standard Procedure for the Determination of Bulk Density and Filter Cake Resistance of Precoat Filter Aids"* was used. The bulk densities reported in Table 5 were determined using a cake thickness in the filter cell of 1.5 to 2.5 in. whereas the standard procedure calls for a cake thickness of 2.5 ± 0.2 in. and the mean of three determinations.

To determine the probable effect of an error in the bulk density of a filter aid on the computed value of the β-index, the results of a typical run (run 91) were used. The data of this run showed an actual filter aid bulk density of 12.76 lbs/ft^3 and a value of the β-index of 1.132×10^7. The value of the β-index was calculated with other values of filter aid bulk density with results as follows:

Trial no.	γ_p assumed, lbs/ft^3	β-index, ft^{-2}	% change from trial 1	
			In γ_p	In β
1	12.00	1.151×10^7	–	–
2	12.40	1.139×10^7	3.33	1.04
3	12.76	1.132×10^7	6.33	1.6
4	15.00	1.085×10^7	25.00	5.7

*This procedure is included as Part IV of the Final Report.

Table 5. In-place bulk density, γ_p, of the various filter aids used in this study.

Serial no.	Filter aid identification	In-place bulk density (lbs/ft^3)
1	E6-2	19.67
2	E6-3	19.36
3	E6-4	19.31
4	E5-1	26.41
5	E5-3	22.54
6	E5-17	22.08
7	E2-1	21.05
8	E2-2	20.97
9	E2-3	21.15
10	E2-4	21.28
11	G4-1	24.59
12	G4-2	23.85
13	G4-3	23.71
14	G4-4	23.54
15	G1-1	22.92
16	G1-2	22.51
17	G1-3	22.58
18	G1-5	23.14
19	J4-4	20.27
20	J4-6	22.54
21	J4-7	20.75
22	J3-5	19.49
23	J3-6	19.49
24	J3-7	19.81
25	J0-1	20.65
26	J0-4	20.40
27	J0-6	21.08
28	S2-1	11.31
29	S2-3	11.04
30	S2-4	11.50
31	S2-5	10.35
32	S2-6	9.69
33	S2-9	10.34
34	S2-20	10.07
35	S2-21	10.62

Table 5. Continued.

Serial no.	Filter aid identification	In-place bulk density (lbs/ft^3)
36	S3-1	12.16
37	S3-2	12.69
38	S3-3	12.76
39	S3-4	13.15
40	S4-1	13.23
41	S4-2	13.47
42	S4-3	12.34

The results indicate that:

- An increased value of the bulk density of a filter aid will provide a lower value of the β-index.
- A 10% error in the actual value of the bulk density of a filter aid will produce about a 3% error in the β-index.

Since the bulk densities reported in Table 5 are accurate within about 5% the values of β reported in this study are accurate within about 1.5 to 2.0%.

Summary of Runs and Determination of β-Indices

Over 300 filtration runs were made in this study. A total of 42 different samples of filter aids were evaluated in the laboratory and at least four filter runs were made using each sample. The data collected in a typical filter run (Run 201) using filter aid E6-2 are listed in Table 6.

The headloss versus time of filtration curve for this run is plotted in Fig. 3. The figure clearly indicates the existence of the period of

Table 6. Data from Run 201.

Clock time	Time since start of run, minutes	Headloss, inches of Hg
15:45	0	0.10
15:56	11	2.30
16:05	20	6.80
16:15	30	11.70
16:25	40	15.90
16:35	50	19.90
16:46	61	23.90
16:55	70	27.00
17:05	80	30.40
17:15	90	33.80
17:25	100	36.70
17:35	110	39.80
17:45	120	42.80
17:55	130	45.40
18:05	140	47.90
18:15	150	50.60

C_S = 7.86 mg/l

C_F = 308.1 mg/l

Filter aid: E6-2

Fe^{+++} concentration in the effluent = < 0.03 mg/l

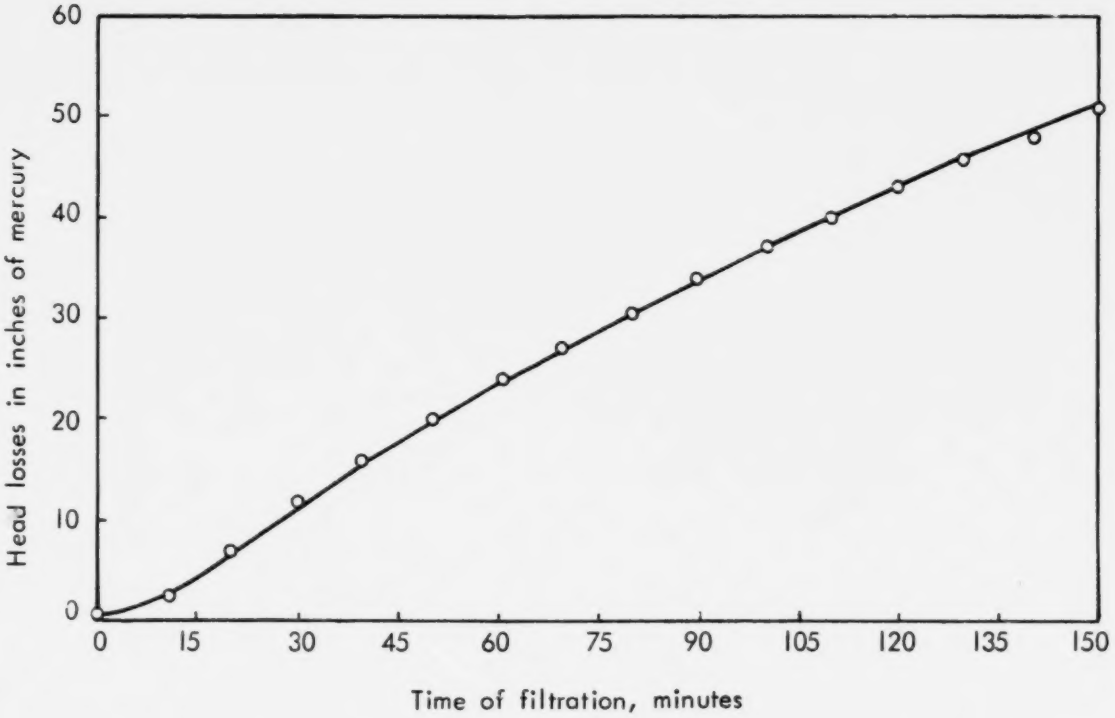

Fig. 3. Headloss versus time of filtration, run 201. C_S = 7.86 ppm; C_F = 308.1 ppm; Filter aid used: E6-2.

initial dilution when a smaller rate of headloss increase exists early in the run. The point of inflection at a filtration time of approximately 10 min corresponds with the end of the initial dilution effect. As described earlier on page 13, the latter portion of the curve shows the increasing area effect. The same type of curve resulted in all the runs. However, runs made with a low concentration of body feed (about 60 to 80 mg/l) did not clearly show the increasing area effect since the latter portion of the curve was nearly a straight line. The data collected in the runs were used to determine the β-indices. Appendix I-B contains an example illustrating the computation of a β-index from the results of a typical run.

The values of the β-indices and the multiple linear correlation coefficients (R) obtained from the results of the filtration runs are listed in Table 15 in Appendix I-C. Table 15 also lists the suspended solids concentration (C_S) in the synthetic water, the body feed concentration (C_D) used in the run and the ratio of C_S/C_D. R is an index of closeness of linear fit between the two variables (i.e., headloss in feet of water and Ln $(1 + R_S \phi x/R_o^2)$ in this case). For a perfect linear relationship between the two variables, R would equal 100%.

Determination of β-Prediction Equations and Test of Their Adequacy for Prediction of Beta

Values of β-indices and C_S/C_F obtained in a series of filtration runs using the same sample of filter aid were used to determine the β-prediction Equation (Eqs. (10) and (11)) for that filter aid. A computer program, "MAIDS," was used for this purpose. The results of filtration runs made with the same filter aid sample are grouped together in Table 15.

The exponents of the β-prediction equations obtained for each sample of filter aid are listed in Table 7. This table also contains the respective values of the multiple linear correlation coefficients and the predicted values of β-indices using the simplified equation (i.e., $\beta = 10^a(C_S/C_F)^b$) at three levels of C_S/C_F: 0.02, 0.05 and 0.1.

Figure 4 is a typical plot of the log (C_S/C_F) versus log β obtained using one filter aid sample (filter aid E5-17, runs 184-187). The relationship observed between the two variables, as is evident from Fig. 4, is a straight line. Similar plots of the data for the other filter aids provided a linear relationship. This substantiates earlier findings [13,18].

Table 7. Summary of β-prediction equations and values of β-indices, predicted at C_S/C_F levels of 0.02, 0.05, and 0.10.

Constants: Source of iron: $FeCl_3$ solution in tap water plus 0.5 mg/ℓ of copper sulfate, as copper.
Effluent Iron Concentration: 0 to 0.06 mg/ℓ
C_S: about 8.0 mg/ℓ (7.5-8.5 range)
C_D: variable between runs
Filter area: 3.20 sq ft

C_S/C_D: ranges from about 0.02 to 0.2 between runs (C_S/C_D from 5 to 50)
Temperature: 60°F
Q: 1 gpm/sq ft

Filter aid identification	Runs	β-prediction equations and values of R (From laboratory test results)							Predicted Values of β, in Ft^{-2}, using β-prediction equation $\beta = 10^a (C_S/C_D)^b$ at different values of C_S/C_D)		
		R	$\beta = 10^a \cdot (C_S/C_D)^b \cdot (C)^c$ a =	b =	c =	R	$\beta = 10^a (C_S/C_D)^b$ a =	b =	C_S/C_D = 0.02 ($\times 10^6$)	C_S/C_D = 0.05 ($\times 10^6$)	C_S/C_D = 0.1 ($\times 10^6$)
S2-4	39-45	99.909	12.98741436	-0.85497481	-3.02242184	99.827	10.3851089	2.18793297	4.625	34.29	157.5
S2-3	46-52	99.786	11.13402271	1.27628231	-0.88177395	99.608	10.45026302	2.24277973	4.469	34.69	161.3
S2-1	53-60	99.844	10.77726841	1.63628960	-0.51455784	99.754	10.33092785	2.15612984	4.68	33.95	149.6
S2-20	61-66	99.288	5.42328548	7.53267097	5.40589237	99.152	10.24094296	2.09768581	4.796	33.18	139.1
S2-5	122-126	99.783	13.02931213	-0.94849741	-3.06680107	99.530	10.27379322	2.13686562	4.506	31.90	137.1
S2-9	127-131	99.794	10.50879574	1.84311581	-0.26402664	99.748	10.26930141	2.10673809	4.890	33.83	145.4
S2-6	132-136	99.756	7.98150253	4.53664684	2.45759678	99.558	10.12129307	2.03666687	4.584	29.60	121.5
S2-21	169-173	99.822	15.85924244	-5.20033073	-6.86884212	99.345	10.13061142	2.00464249	5.443	33.93	135.7
S3-4	29-38	99.575	10.52720261	1.73644829	-0.28884602	99.566	10.30090046	2.04407120	6.621	43.74	180.6
S3-2	73-78	99.475	10.29731274	1.81636715	-0.13136101	99.492	10.16797256	1.93898773	7.550	44.45	169.3
S3-1	79-86	99.285	11.24680042	0.98341548	-1.05665302	99.115	10.34136105	2.06498718	6.781	45.32	189.0
S3-3	87-92	99.868	11.12158816	0.74192983	-1.14020252	99.721	10.20643139	1.94099712	8.061	48.01	184.3
S4-3	111-115	99.954	11.75993252	0.73294222	-1.43278217	99.966	10.50884056	2.20301819	6.772	44.5	202.2
S4-2	116-121	99.522	15.31389523	-4.12016678	-5.88876820	99.169	10.29753590	1.97551537	8.861	53.0	209.9
S4-1	179-183	99.761	2.62629986	10.35785007	8.38543415	99.301	10.23429012	1.93304539	9.000	52.04	200.1
J4-6	107-110	99.973	9.16131401	3.05229187	1.10412025	99.964	10.21306419	1.97430611	7.212	43.96	173.3
J4-4	152-156	99.856	11.59778118	0.80631971	-1.28725147	99.591	10.39118481	2.06353569	7.730	51.42	212.4
J4-7	260-264	99.929	13.37145615	-1.83411789	-3.67333317	99.894	10.04663086	1.80446148	9.583	50.12	174.7
J3-5	93-97	99.975	10.57284260	1.40189075	-0.49469755	99.955	10.14858437	1.90635872	8.103	48.61	174.7
J3-6	98-102	99.443	10.07520294	2.36152458	0.32397366	99.408	10.29101086	1.98994541	8.147	50.82	200.0
J3-7	164-168	100.000	13.72517300	-1.62155533	-3.67566967	99.704	10.30087757	1.97055626	9.034	55.05	213.9
J0-4	103-106	100.000	11.09186363	1.73551178	-0.46211338	99.986	10.68011379	2.20969772	8.320	63.11	295.4
J0-1	159-163	100.000	12.46984577	-0.62650388	-2.48561573	99.933	10.23762984	1.88322449	10.90	61.32	226.1
J0-6	265-269	99.872	13.97296619	-1.75044441	-3.85347652	99.715	10.59562193	2.12386036	9.806	68.18	296.2
E6-4	188-191	99.968	10.71950626	1.87823772	-0.24578762	99.975	10.50224018	2.12663460	7.802	54.61	237.4
E6-2	201-204	99.906	12.82551670	-0.91011024	-2.86617184	99.827	10.47600460	2.09503746	8.241	56.36	240.4
E6-3	256-259	99.705	16.08139038	-4.01337624	-6.11972234	99.348	10.59508705	2.15499687	8.611	62.09	275.4
E5-1	137-141	100.000	10.31914234	2.26166344	0.26898193	100.000	10.51323986	1.96711254	14.90	91.87	351.7
E5-17	184-187	99.957	12.63197327	-1.04734612	-2.83272552	99.841	10.31385231	1.92853832	10.93	62.20	242.9
E5-3	197-200	99.890	12.48021412	-0.30677181	-2.28769398	99.863	10.45404720	1.98954678	11.86	73.97	291.4
E2-4	192-196	100.000	11.24984550	1.19025707	-0.80966377	99.952	10.56048584	2.02056885	13.50	86.19	346.6
E2-1	205-208	99.894	13.62593174	-1.49916363	-3.52555656	99.759	10.51301670	2.00597382	12.97	81.45	321.4
E2-3	220-224	99.984	12.18756104	0.12043186	-1.86335564	99.968	10.49256706	1.97578430	13.59	83.66	328.7
E2-2	270-275	99.947	12.29683590	-0.02055234	-2.00966655	99.895	10.57424831	2.03129101	13.31	85.94	349.0
G4-1	142-146	99.840	10.43643369	1.83071041	-0.08190536	99.840	10.36339092	1.91228580	13.28	76.10	282.6
G4-2	209-213	99.901	10.03252029	2.46023083	0.46097279	99.908	10.48875809	2.02904510	10.98	70.58	288.2
G4-5	226-229	99.990	10.58006477	1.64126110	-0.28952312	99.980	10.32860470	1.93623259	11.01	64.36	246.8
G4-4	231-235	99.956	10.73429871	1.34741688	-0.53086472	99.934	10.28011227	1.89345551	11.54	65.77	243.6
G4-3	236-240	99.810	8.65023391	3.84545612	1.88726139	99.814	10.40164185	1.99196053	10.46	64.86	256.8
G1-1	147-151	99.611	11.29054165	0.59536501	-1.25476742	99.430	10.19460106	1.77184582	15.47	78.45	264.7
G1-2	214-219	99.825	9.9545850	2.26117706	0.43965054	99.825	10.35547256	1.83100605	17.76	94.51	334.5
G1-3	251-255	99.801	11.97352219	0.09870034	-1.78785324	99.536	10.48374462	1.97423553	13.36	81.48	323.3

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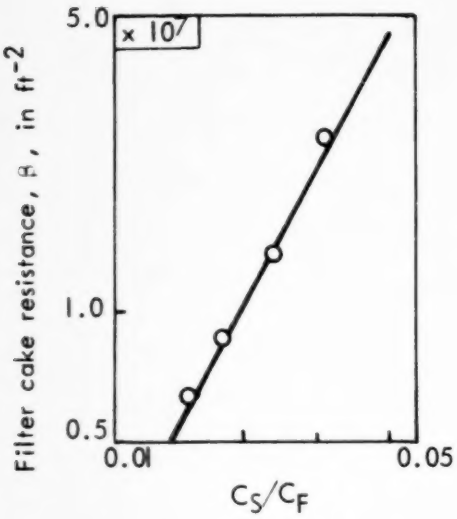

Fig. 4. Filter cake resistance (β) versus ratio of suspended solids concentration to body feed concentration (C_S/C_F). Filter aid: E5-17 Runs: 184-187.

The value of the linear correlation coefficient obtained in analyzing the results obtained using each filter aid tested in this study is generally over 99%. This is excellent correlation considering that this value is an outcome of experimental work requiring control of various factors such as temperature, flow rate, drip rates of chemicals, etc.

The values of the linear correlation coefficients obtained using Eq. (10) is generally higher than those obtained using Eq. (11). Exceptions are noted when using filter aid samples S3-2, S4-3, E2-2, E5-1, E6-4, G4-2 and G1-2. By and large, no significant difference exists in the values of R with the two types of equations. It was concluded, therefore, that there is no special advantage gained in adding the additional variable C_F to the equation when dealing with iron-bearing waters. This is in agreement with earlier findings[18]. The simplified form of the equation ($\beta = 10^a (C_S/C_F)^b$) was, therefore, used for β prediction.

A study of Table 15 reveals that nearly all of the filtration runs were conducted using an approximate range in the values of C_S/C_D from 0.02

to 0.10. In a few cases, however, this range has not been fully covered (the data in Fig. 4, for example) and plots of log C_S/C_F versus log β need extrapolation to cover this range. Since a straight line relationship has been established between these two variables, such extrapolation is reasonable in order to compare the filter aids. Excellent values of R add to the confidence in extrapolating the observed data.

To demonstrate the accuracy of the β-prediction equations, plots of observed and predicted values of β can be compared. (See Fig. 5.)

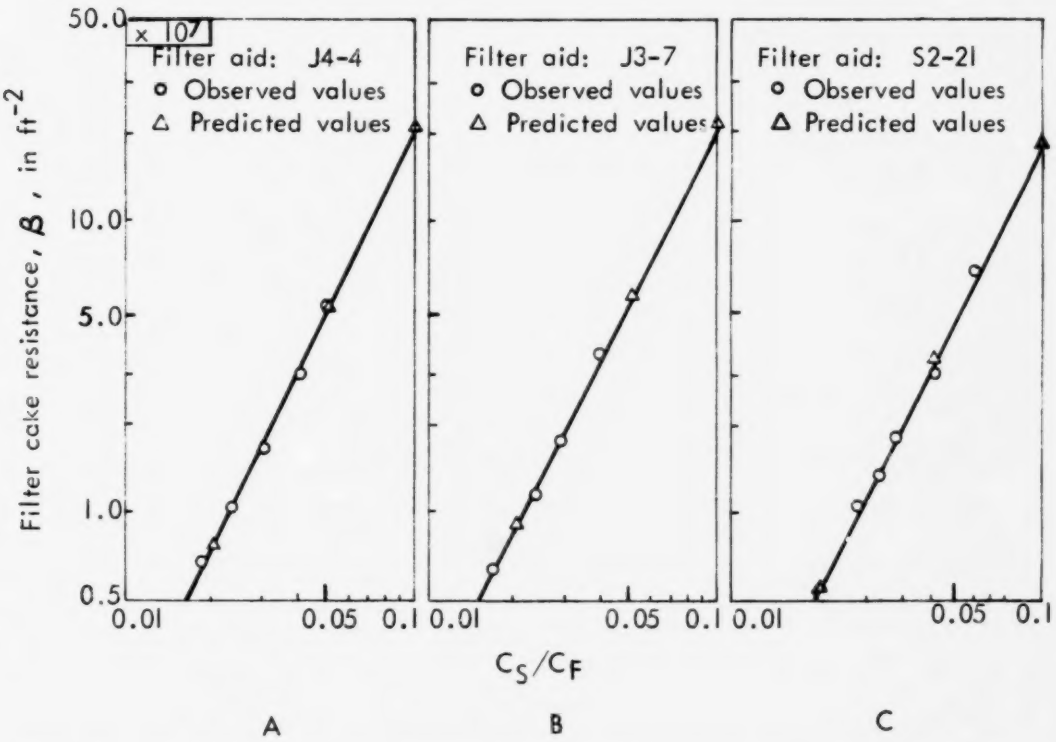

Fig. 5. Typical plots of observed (from filter run results) and predicted (from β-prediction equation) values of filter cake resistance, β, versus C_S/C_F for the filter aids shown.

A	J4-4	Runs 152-156
B	J3-7	Runs 164-168
C	S2-21	Runs 169-173

Three filter aid samples (J4-4, J3-7 and S2-21) were chosen arbitrarily for comparison of observed and predicted values of β. Figure 5 shows that one single line passes through the points plotted with observed values of β and those plotted with predicted values of β. This demonstrates the adequacy of prediction equations for β prediction. Accordingly, β indices were computed corresponding to C_S/C_F levels of 0.02, 0.05 and 0.1 (Table 7) and used to compare filter aid hydraulic characteristics.

In determining the β-prediction equation representative of filter aid S3-4, a total of nine separate filter runs (runs 29-38, Table 15) were made. The results of these runs were used to evaluate the probable accuracy of β-prediction when the prediction equation is based on only 4 to 5 runs instead of 8 to 10 runs. A maximum of 10 runs could be made using a single bag of filter aid. In making the evaluation, the ten filter runs were divided into six subgroups.

Subgroup	Runs and (C_S/C_F value)				
1	30(0.0575),	31(0.0595),	32(0.0737),	36(0.0257),	38(0.0184)
2	29(0.0605),	33(0.1045),	34(0.0286),	35(0.0407),	37(0.0248)
3	30(0.0575),	32(0.0737),	34(0.0286),	36(0.0257),	38(0.0184)
4	29(0.0605),	31(0.0595),	33(0.1045),	35(0.0407),	37(0.0248)
5	30(0.0575),	31(0.0595),	34(0.0286),	35(0.0407),	36(0.0257)
6	29(0.0605),	32(0.0737),	33(0.1045),	37(0.0248),	38(0.0184)

The subgroupings were made at random (subgroup 5), in order of runs (odd numbers – subgroup 4, and even numbers – subgroup 3), and to cover the widest ranges of (C_S/C_F) as in subgroups 1 and 2.

The data from each subgroup of runs were used to determine the β-prediction equation for that group. These equations are listed in Table 8.

Table 8. Predicted filter cake resistance, β, at indicated levels of C_S/C_F using data of subgroups for filter aid S3-4.

Runs	[a]Values of exponents a =	b =	R in percent	Values of $\beta/10^6$ ft^{-2} at different C_S/C_D levels C_S/C_D = 0.02	C_S/C_D = 0.05	C_S/C_D = 0.1
29-38	10.300900	2.044071	99.57	6.621 (0)[b]	43.74 (0)	180.6 (0)
30, 31, 32, 36, 38 (subgroup 1)	10.213853	1.975564	99.94	7.201 (+ 8.8)	44.0 (+ 0.6)	173.1 (- 4.2)
29, 33, 34, 35, 37 (subgroup 2)	10.397864	2.121810	99.58	6.201 (- 6.3)	43.38 (- 0.8)	188.8 (+ 4.5)
30, 32, 34, 36, 38 (subgroup 3)	10.249131	1.998209	99.74	7.148 (+ 8.0)	44.61 (+ 2.0)	178.2 (- 1.3)
29, 31, 33, 35, 37 (subgroup 4)	10.427476	2.154083	99.59	5.857 (- 11.5)	42.16 (- 3.6)	187.7 (+ 3.9)
30, 31, 34, 35, 36 (subgroup 5)	10.122759	1.928028	99.97	7.033 (+ 6.2)	41.14 (- 6.0)	156.6 (- 13.3)
29, 32, 33, 37, 38 (subgroup 6)	10.342592	2.063381	99.59	6.869 (+ 3.8)	45.50 (+ 4.0)	190.2 (+ 5.2)

[a] $\beta = 10^a(C_S/C_D)^b$.

[b] Values within brackets are the percent differences as compared to the respective computed values using the data of runs 29-38.

Then, β- ndices are calculated at the (C_S/C_F) levels of 0.02, 0.05, and 0.10 using these equations and are also listed in Table 8. The most representative β-prediction equation, of course, is that obtained by an analysis of the data from all of the runs together. This β-prediction equation (runs 29 through 38) is also included in the table.

Table 8 also shows the percentage difference between the values of β at (C_S/C_F) levels of 0.02, 0.05, and 0.10 calculated using the β-prediction equations for each subgroup and the β-prediction equation representative of all the runs. In general, the percentage variations did not exceed 10%. The least accurate prediction equation (subgroup 5) was predicted from runs covering the relatively narrow range of (C_S/C_F) from 0.0257 to 0.0595. This analysis indicates that the values of β-indices predicted from the β-prediction equations developed and used in the study are generally accurate within $\pm$ 10%. A similar analysis using the results of runs (79-86) for filter aid S3-1 led to a similar conclusion.

Comparison of Filter Aids

From the foregoing paragraphs it is reasonable to base a comparison of the resistance of the filter cakes resulting from the use of different filter aid samples on the values of β predicted from the equations developed in this study instead of on the values actually observed. Such comparisons should be made, however, only when the filter cakes are formed under identical conditions. Such conditions exist at equal values of C_S/C_F. Therefore, values of β have been computed using the simplified prediction equation at three levels of C_S/C_D: 0.02, 0.05 and 0.1. The results are tabulated in Table 7.

Figures 6 through 9 show the range of the predicted filter cake resistance (β) at values of C_S/C_F of 0.02, 0.05 and 0.10 for all the various samples of filter aid tested in this study. The range has been shown in preference to independent regression lines for each sample of filter aid since this avoids confusion which would result by the plotting of

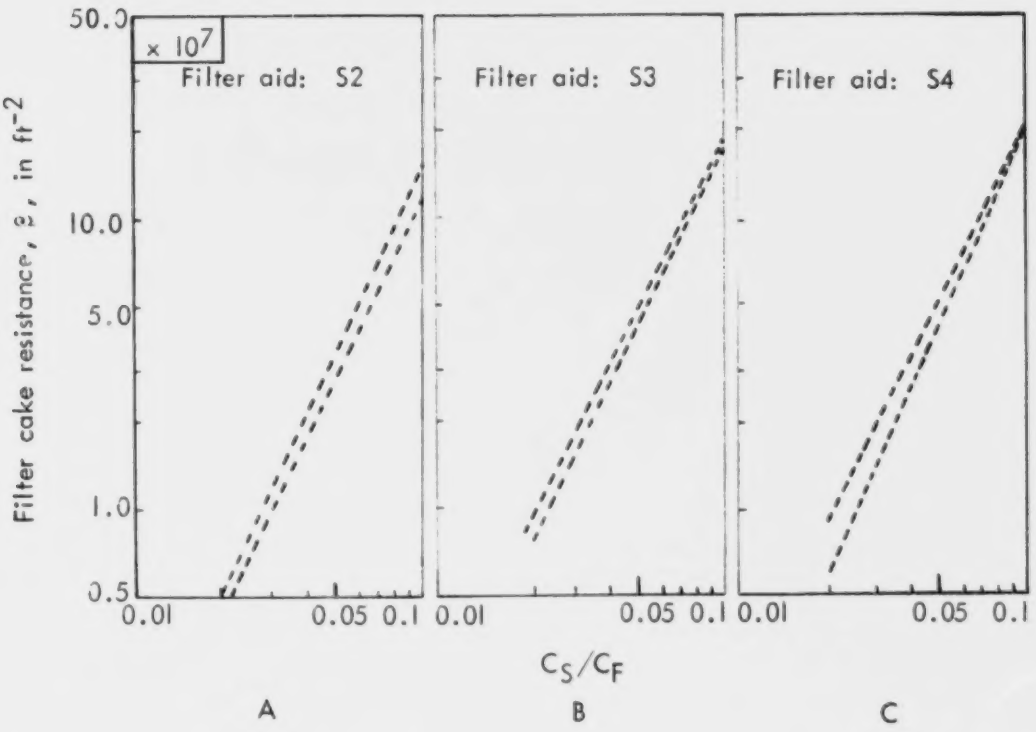

Fig. 6. Range of values of filter cake resistance, β, predicted at C_S/C_F values of 0.02, 0.05 and 0.10 for filter aids shown
A S2 B S3 C S4.

overlapping regression lines. It seems more meaningful to compare the filter aids on the basis of both the range of the regression lines and the range of the values of β indices listed in Tables 9 to 11.

The analysis of the data collected in the runs and the comparison of filter aids on the basis of range regression lines (Figs. 6-9) and on the basis of the tabulated values of β-indices (Tables 9 to 11) reveal that:

1. All of the filter aids used in this study removed iron from water practically to the same extent over the entire body feed range used from about 80 mg/l to 500 mg/l. The effluent

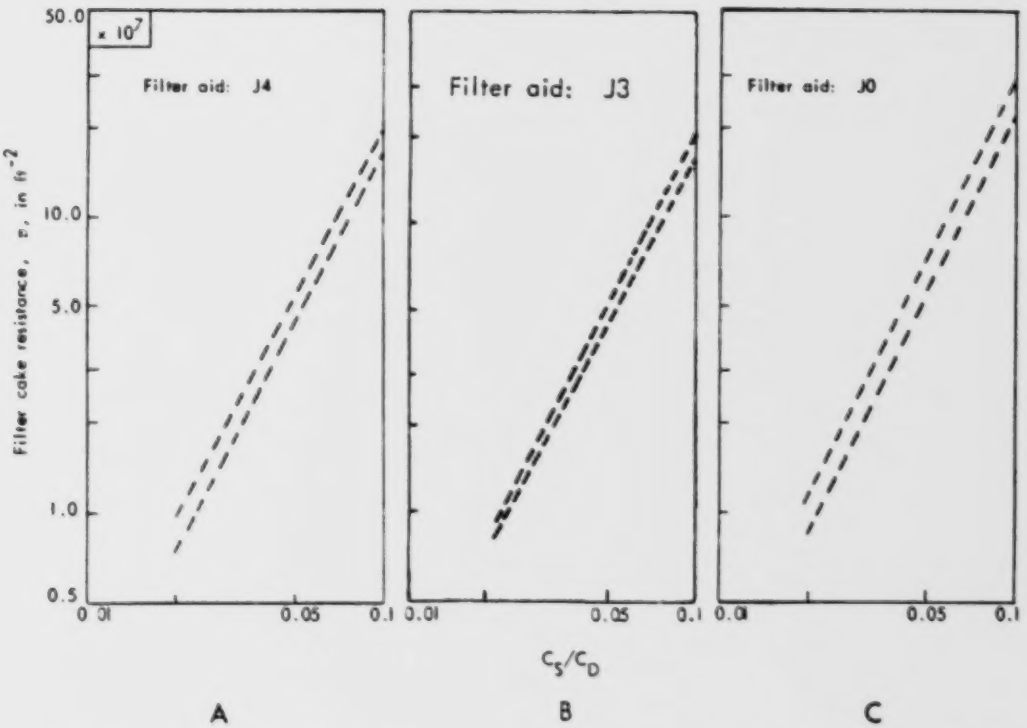

Fig. 7. Range of values of filter cake resistance, β, predicted at C_S/C_F values of 0.02, 0.05 and 0.10 for filter aids shown
A E6 B E5 C E2.

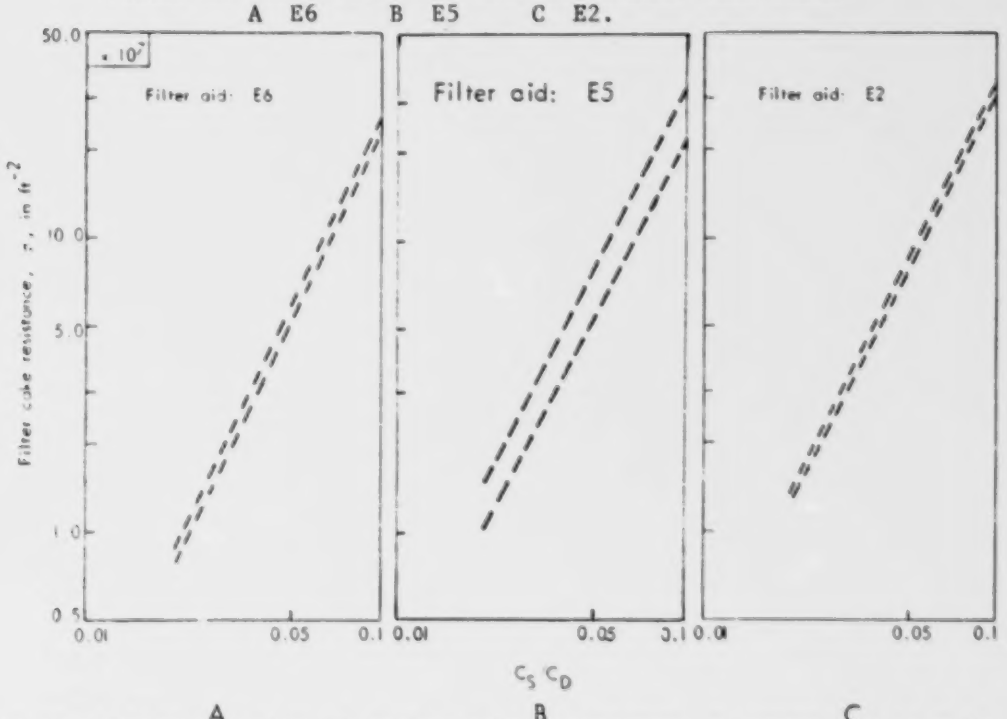

Fig. 8. Range of value of filter cake resistance, β, predicted at C_S/C_F values of 0.02, 0.05 and 0.10 for filter aids shown
A E6 B E5 C E2.

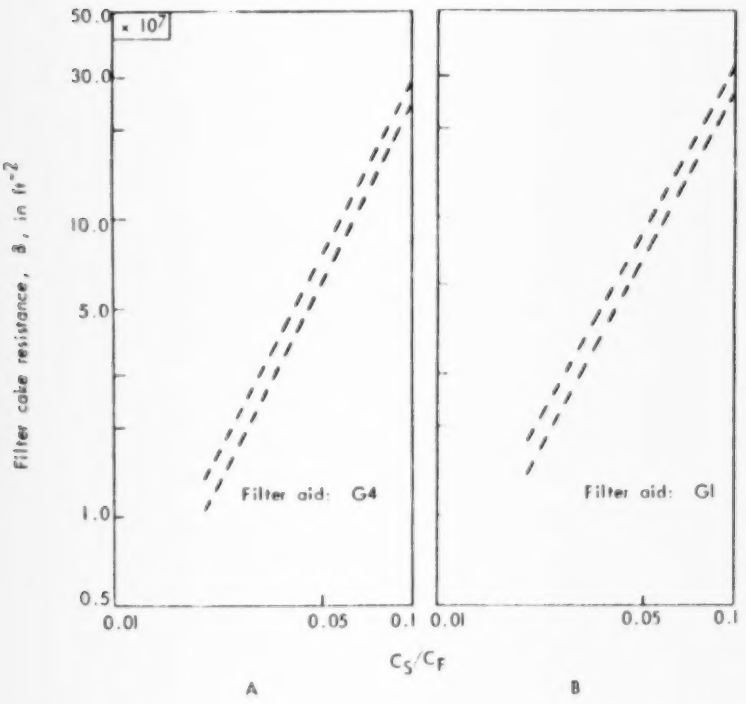

Fig. 9. Range of values of filter cake resistance, β, predicted at C_S/C_F values of 0.02, 0.05 and 0.10 for filter aids shown A G4 B G1.

in all the filter runs contained from 0 to 0.06 mg/l of iron irrespective of the type, grade, and quantity of filter aid used as body feed (range 80-500 mg/l).

2. Under identical conditions, the filter cake resistance, β, is lower for S2 than for any other filter aid used in this study. For example, at C_S/C_F level of 0.02, Table 9 shows an average value of β for S2 of $4.750 \times 10^6/ft^2$. The values of β for all the other filter aids are higher than $4.750 \times 10^6/ft^2$. Similar results are observed at other values of C_S/C_F in Tables 10 and 11.

Table 9. Computation of confidence limits for values of β-indices predicted at C_S/C_F level of 0.02 using β-prediction equations.

	S2	S3	S4	J4	J3	J0	E6	E5	E2	G4	G1
β values(a)	4.625 4.469 4.687 4.796 4.506 4.890 4.584 5.441	7.550 6.781 8.061 6.621	9.000 5.872 8.861	7.212 7.730 9.585	8.100 8.147 9.037	8.320 10.900 9.806	7.802 8.241 8.611	14.90 10.93 11.86	13.50 12.97 13.59 13.31	13.28 10.98 11.01 11.54 10.46	15.47 17.76 13.36
n =	8	4	3	3	3	3	3	3	4	5	3
ΣX	37.998	29.013	23.733	24.527	25.284	29.029	24.654	37.690	53.370	57.270	46.590
$\bar{X}$	4.750	7.253	7.911	8.172	8.427	9.675	8.218	12.56	13.34	11.45	15.53
Σx^2	0.663	1.377	0.243	0.293	0.553	3.373	0.327	8.874	0.493	5.208	9.69
s^2	0.095	0.459	0.123	0.147	0.277	1.687	0.164	4.437	0.164	1.302	4.84
$s_{\bar{x}}$	0.109	0.339	1.02	0.74	0.304	0.75	0.235	1.215	0.202	0.51	1.27
Upper limit C.L.95	5.01	8.33	12.30	11.35	9.73	12.90	9.23	17.78	13.98	12.87	20.99
Lower limit	an 4.49	and 6.18	and 3.53	and 4.99	and 7.12	and 6.46	and 7.20	and 7.34	and 12.70	and 10.03	and 10.07

(a) All values have been divided by 10^6 in order to save space and simplify calculations. The correct values for n, ΣX, $\bar{X}$, $S_{\bar{x}}$ and confidence limits can be obtained by multiplying the values in the table by 10^6. However, Σx^2 and S^2 can be obtained by multiplying the respective values by 10^{12}.

3. There is relatively little difference between the range of the regression lines for S3 and S4 (Fig. 6) indicating that these two grades of Sil-Flo develop nearly the same filter cake resistance at the same C_S/C_F level. This is confirmed in the results shown in Table 9 since the average ($\bar{x}$) β values for S3 and S4 are $7.253 \times 10^6/ft^2$ and $7.911 \times 10^6/ft^2$, respectively.

4. The Celite filter aid J4 produces a slightly higher filter cake resistance than that produced by the Sil-Flo filter aid S3. For example, at a C_S/C_F level of 0.02, the average ($\bar{x}$) β values for these two filter aids are $8.172 \times 10^6/ft^2$ and $7.253 \times 10^6/ft^2$, respectively. Similarly, filter aid J3 produced a slightly

Table 10. Computation of confidence limits for values of β-indices predicted at C_S/C_F level of 0.05 using β-prediction equations.

	S2	S3	S4	J4	J3	J0	E6	E5	E2	G4	G1
β-values[a]	34.29 34.69 33.95 33.18 31.90 33.83 29.60 33.93	44.43 45.32 48.01 43.74	52.04 44.34 53.39	43.96 51.42 50.12	48.61 50.82 55.05	63.11 61.32 68.18	54.61 56.36 62.09	91.87 62.20 73.97	86.19 81.45 83.66 85.94	76.10 70.58 64.36 65.77 64.86	78.45 94.51 81.48
n =	8	4	3	3	3	3	3	3	4	5	3
ΣX	265.37	181.52	149.97	145.50	152.48	192.81	173.06	228.04	337.24	341.67	254.44
$\bar{X}$	33.171	45.380	49.990	48.500	50.830	64.200	57.690	76.010	84.310	68.330	84.810
Σx^2	19.678	10.473	45.466	31.762	37.650	26.606	29.462	447.938	16.424	102.523	147.325
S^2	2.811	3.491	22.733	15.881	18.825	13.303	14.731	223.969	5.475	25.631	73.662
$S_{\bar{X}}$	0.59	0.94	2.76	2.30	2.50	2.10	2.21	8.65	1.17	2.26	4.95
Upper limit C.I.95 Lower limit	34.57 and 31.77	48.35 and 42.41	61.86 and 38.12	58.40 and 38.60	61.57 and 40.07	73.22 and 54.18	67.19 and 48.19	112.31 and 39.71	88.03 and 80.59	74.63 and 62.03	106.11 and 63.51

[a] All values have been divided by 10^6 in order to save space and simplify calculations. The correct values for $\bar{x}$, ΣX, $\bar{X}$, $S_{\bar{X}}$ and confidence limits can be obtained by multiplying the values in the table by 10^6. However, Σx^2 and S^2 can be obtained by multiplying the respective values by 10^{12}.

higher filter cake resistance than filter aid J4 (Tables 9 to 11). Filter aid J0 produced still higher values of filter cake resistance.

5. The Eagle-Picher grades (E6, E5 and E2) produced higher filter cake resistance than both the Sil-Flo and Celite grades. Filter aid E6 produced cake resistance lower than E5, E2 produced filter cake resistance higher than E5. For example, at a C_S/C_F level of 0.02, the mean values of β-indices are:

$$E6 - 8.218 \times 10^6/ft^2$$

$$E5 - 12.560 \times 10^6/ft^2$$

$$E2 - 13.34 \times 10^6/ft^2.$$

Table 11. Computation of confidence limits for values of β-indices predicted at C_S/C_F level of 0.10 using β-prediction equations.

	S2	S3	S4	J4	J3	J0	E6	E5	E2	G4	G1
Values of β	157.5 161.3 149.6 139.1 137.1 145.4 121.5 133.7	169.3 189.0 184.3 180.6	200.1 202.2 209.9	173.3 212.4 174.7	174.7 220.3 213.9	241.4 216.1 296.2	237.4 240.4 275.4	351.7 242.9 291.4	346.6 321.4 328.7 349.0	282.6 288.2 246.8 243.6 256.8	264.7 334.7 323.3
n =	8	4	3	3	3	3	2	3	4	5	3
Σx	1145.2	724.2	612.2	560.4	588.6	817.7	753.2	886.0	1345.7	1318.0	922.7
$\bar{x}$	143.15	180.80	204.10	186.80	196.20	272.60	251.10	295.30	336.40	263.60	307.60
Σx^2	1191.04	211.8	53.20	98.4	789.98	3184.53	842.95	6000.99	814.37	1694.64	2625.95
S^2	170.43	70.59	26.60	492.01	399.99	1592.26	421.22	3000.49	204.79	423.66	1312.97
$S_{\bar{x}}$	4.6	4.2	3.0	12.8	11.5	23.0	11.9	31.6	7.2	9.2	20.9
Upper limit C.L.95 and Lower limit	154.1 and 132.2	194.2 and 167.4	216.9 and 191.3	241.8 and 131.8	245.6 and 146.8	371.6 and 173.6	302.1 and 200.1	431.2 and 159.4	359.2 and 313.6	289.2 and 238.0	397.6 and 217.6

(a) All values have been divided by 10^6 in order to save space and simplify calculations. The correct values for $\bar{x}$, Σx, $\bar{x}$, $S_{\bar{x}}$ and confidence limits can be obtained by multiplying the values in the table by 10^6. However, Σx^2 and S^2 can be obtained by multiplying the respective values by 10^{12}.

6. The Dicalite grade G1 produced the highest cake resistance of all the filter aids tested. The G4 grade also generally produced higher filter cake resistance than that produced by the other filter aids. The respective mean values of β-indices were 11.45 x $10^6/ft^2$ for G4 and 15.53 x $10^6/ft^2$ for G1.

The manufacturer of Sil-Flo filter aids prepared a table showing the grades of diatomite filter aids approximately equivalent to their own perlite grades when used on an equivalent volume basis (Appendix I-D). This comparative table indicates that the Sil-Flo Corp. considers that volumetrically the comparative grades from different manufacturers are as follows:

Sil-Flo grades	Eagle Picher grades	Johns-Manville grades	Great Lakes carbon grades
S2	E6	J4	G4
S3	E2	J0	G1

In order to establish the validity of this table, the values of the β-indices for all the filter aids were also compared on the basis of equal volumes of body feed.

The comparison was made at an arbitrarily selected C_S/C_F level of 0.05 for the S2 filter aid. The β for each of the filter aids was calculated when the cake formed with that filter aid contained the same volume of filter aid as that contained in the S2 filter cake at the C_S/C_F level of 0.05. This conversion was made by using the mean bulk density data in Table 5 for the respective grades. At the equal volume of filter aid levels of C_S/C_F calculated for each filter aid type and grade, β-indices were calculated using the appropriate representative β-prediction equation*, listed in Table 12. The values of bulk density, the equivalent levels C_S/C_F and β-indices for all the filter aids are:

*These β prediction equations are developed in the section "Discussion of Results."

Filter aid designation	Cake bulk density[a] lbs/ft^3	C_S/C_F[b]	$\beta/10^6$, ft^{-2}
S2	10.61	0.0500	32.73
S3	12.69	0.0418	31.92
S4	13.01	0.0408	33.11
J4	21.19	0.0250	10.23
J3	19.56	0.0271	15.14
J0	21.08	0.0251	15.31
E6	19.45	0.0273	15.78
E5	23.68	0.0224	15.38
E2	21.11	0.0251	21.63
G4	23.77	0.0223	14.19
G1	22.67	0.0234	20.65

[a]Mean values for a filter aid grade, Table 5.

[b]Level of (C_S/C_F) to provide equal volumes of filter aid in the filter cake.

The comparative table would then provide respective β values as follows:

Group 1		Group 2	
Filter aid	$\beta/10^6$	Filter aid	$\beta/10^6$
S2	32.73	S3	31.92
E6	15.78	E2	21.63
J4	10.23	J0	15.31
G4	14.19	G1	20.65

These data, based on an evaluation of the resistance per unit weight of body feed, indicate that:

- The respective grades are not comparable on an equivalent volume basis.
- The diatomite filter aids produce lower β filter cake resistance than perlites when used on an equivalent volume basis. Filter

aid J4 produced the least filter cake resistance and Sil-Flo 443 produced the highest filter cake resistance.

An inspection of the filtration equations in Table 3, Eq. (7), for example, indicates that filter aids will provide comparable performance (same headloss vs time relationship during a filter run) only when they have equal values of βC_F. If equal volumes of filter aid are fed, the values of βC_F calculated for each would be:

Filter aid designation	$(C_S/C_F)^a$	$C_F{}^b$, mg/l	$\beta/10^6$, ft^{-2}	$\beta C_F/10^8$
S2	0.0500	160	32.73	52.4
S3	0.0418	191	31.92	61.0
S4	0.0408	196	33.11	64.9
J4	0.0250	320	10.23	32.7
J3	0.0271	295	15.14	44.7
J0	0.0251	319	15.31	48.8
E6	0.0273	293	15.78	46.2
E5	0.0224	357	15.38	54.9
E2	0.0251	319	21.63	69.0
G4	0.0223	359	14.19	50.9
G1	0.0234	342	20.65	70.6

[a] Level of (C_S/C_F) to provide equal volumes of filter aid in the filter cake.

[b] C_F when C_S = 8 mg/l.

As pointed out earlier, the best filter aid which produces a satisfactory quality of water is neither the one that develops the lowest filter cake resistance, as measured by either β or βC_F, on an equivalent weight basis, nor the one which produces the lowest filter cake resistance on an equivalent volume basis. What is desired is the filter aid that produces an acceptable quality of water at the least cost. The cost concept will be discussed in greater detail in the next section.

The above conclusions are valid but should be considered in light of the limitations imposed on the data evaluation by the unequal sample* sizes for the different filter aids used in the tests. Eight bags of S2 were used in filtration tests but only three to five bags of each of the remaining filter aids were used in the filtration studies. This difference in the size of samples does not invalidate factual observations although it makes the interpretation and the drawing of conclusions based on the factual results more difficult.

With the larger sample size of S2, the observed values of β tend to concentrate around the actual mean value of β for this filter aid. A larger sample, however, also increases the probability of outliers being picked up. In spite of this, none of the values of β-indices for this grade of perlite at any of the three levels of C_S/C_F is larger than the corresponding values for the other filter aids.

Not all bags of this filter aid came from the same source. Filter aids S2-1, S2-3 and S2-4 were received from a single production lot. Similarly, filter aids S2-5, S2-6, S2-9 and S2-20 came from another production lot. Filter aid S2-21 was received from the US Army Engineer Research and Development Laboratories. In spite of the different sources and production lots, this material has consistently developed the least filter cake resistance as measured by its β-index, at equal levels of C_S/C_F.

Statistical tests are good tools for use in developing comparisons as these can make an allowance for the fact that the sample sizes were

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*Hereafter, the sample means a statistical sample. For example, the sample size of the filter aid S2 is 8, that of the filter aid S3 is 4 and so on.

unequal. In other words, statistical concepts would consider that a bigger sample for 10, for example, might have contained a bag of filter aid that would produce a different filter cake resistance than those already observed. Although statistical tests are helpful in minimizing the effects of the difference in sample size, statistical tests are by no means intended to replace good sampling and experimental techniques. Statistical tests were, however, also applied to the results of this study.

Statistical Analysis[24,25]

Essentially, two statistical approaches were used in analyzing the results obtained in this study to:

- Develop confidence intervals for the values of β-indices at three levels of C_S/C_D: 0.02, 0.05 and 0.1.
- Test for homogeneity of means.

Since all the means (i.e., β indices) were unequal as a result of these tests, multiple mean comparisons were also made to determine whether the mean β indices for two or more samples were statistically equal (Appendix I-E). All the statistical tests were conducted at a 95% confidence level.

The confidence intervals calculated for the mean values of the β indices are tabulated in Tables 9-11 and are plotted in Fig. 10. Appendix I-E contains a brief discussion of the basis for these tests and the related calculations.

The results of the comparison of filter aids based on the results of the statistical analyses are essentially the same as those obtained from the comparison of the range in regression lines (Figs. 6-9).

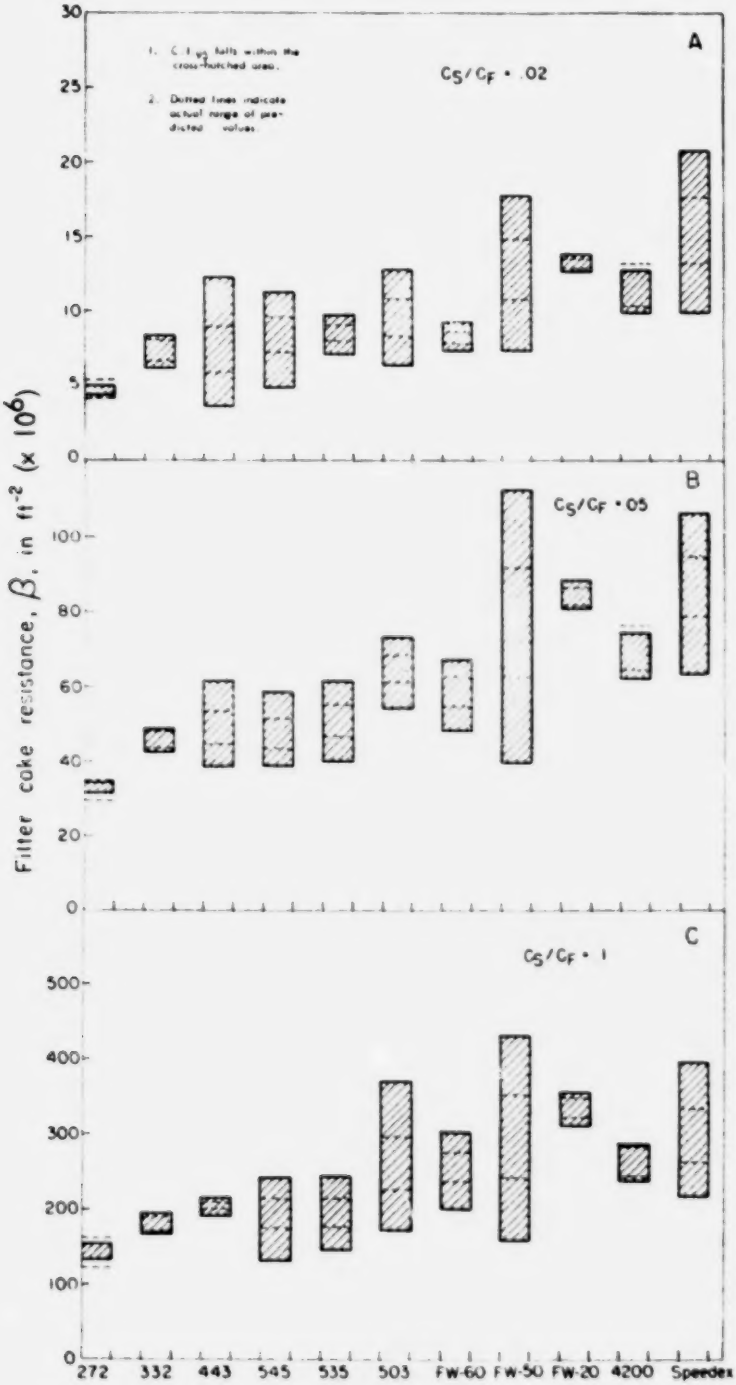

Fig. 10. CI_{95} for filter cake resistance, β, predicted for several filter aids at values of C_S/C_F of
A 0.02 B 0.05 C 0.10.

A comparison of the confidence limits on the value of the β indices for the various filter aids leads to the conclusion that filter aid S2 produces consistently the least filter cake resistance at all three levels of C_S/C_F. The earlier conclusion that the Sil-Flo filter aid grades generally developed less filter cake resistance than the corresponding grades of other filter aids at the same C_S/C_F level is also confirmed by a comparison of the results in Fig. 10. Since the weight of all filter aids fed as body feed at any given level of CS/CF would be the same, the filter aid which has the lowest value of β would also have the lowest value of βC_F, since C_F would be constant for all filter aids. This comparison also indicates that Celite grades produce less cake resistance than corresponding grades of Eagle Picher products. Great Lakes Carbon products developed the highest cake resistance when used on an equivalent weight basis.

Results of the multiple mean tests shown in Tables 16, 17 and 18 in Appendix I-E also corroborate the above conclusions. In addition, these tests indicate that certain filter aids produced filter cake resistance that was not significantly different, on the basis of the test results available, from that produced by several other filter aids. Based on these statistical analyses, the filter aids were grouped together so that those within the same group have filter cake resistances (as measured by either β or βC_F) which are not significantly different from the other filter aids within the group. These groups are listed below in an increasing order of filter cake resistance when fed on an <u>equal weight basis</u>:

1. Sil-Flo 272
2. Sil-Flo 332, Sil-Flo 443, Celite 545 and Celite 535
3. Celite 503 and FW-60

4. FW-50 and Dicalite 4200

5. FW-20, Speedex.

The tests of "homogeneity of means" described in Appendix I-E were also conducted on different production lots of filter aid S2 to see if the materials representing different production lots produced significantly different filter cake resistance (see pages 104-108 in the Appendix I-E). Three statistical samples were considered, one consisting of all eight S2 filter aids tested and the others containing three (S2-1, S2-3, S2-4) and four (S2-5, S2-6, S2-9, S2-20) filter aids, respectively. Filter aid S2-21 was omitted for this analysis since it was a single sample received from a different source and thus, tests of significance were not possible. This analysis demonstrated that there is no significant difference between the mean filter cake resistance of the filter aids representing the different production lots.

DISCUSSION OF RESULTS

Equivalent Performance

In considering the purchase of diatomite or perlite filter aids, two or more may be considered equivalent when they provide the same quality of filtered water with the same filter cake resistance. The quality of water produced by the different types and grades of filter aids available is influenced by the characteristics of the suspended solids to be removed and by the characteristics of the filter aid used in filtration. The quality of the filtered water will improve with the use of finer and finer grades of filter aid, but the filter cake resistance increases when equal weights of the finer filter aids are used as body feed. In practice, it is customary to use the coarsest grade of filter aid (lowest β on an equivalent weight basis) that will provide the desired filtered water quality and to adjust the body feed to obtain the desired filter run characteristics (length of run at a given terminal headloss).

Once a grade of filter aid is selected as satisfactory for a given filtration application, it is frequently necessary to establish its characteristics so that a purchase specification can be written. The specification should be written so that other grades of the same or different materials may be considered for purchase if they provide "equivalent performance." "Equivalent performance" implies that filtration with the equivalent filter aids will provide the same or a better quality of water with the same filter run characteristics as those obtained with the filter aid chosen as the basis of performance. Such performance with the same type of material, for example, can be obtained when using finer grades of filter aid by increasing the rate of body feed. These

finer grades of a filter aid are usually slightly cheaper than the coarser grades. The use of increased amounts of the finer grades may provide "equivalent performance" which would be acceptable to a purchaser if the cost of purchasing and shipping a greater amount of material to the point of use is comparable to that of the filter aid chosen as the basis of the specification.

Two or more filter aids may be considered "equivalent" when they provide "equivalent performance." Equivalence of filter aids which produce the desired quality of effluent may be judged quantitatively on a number of different bases:

1. Filter aids can be considered equivalent if they produce the same unit filter cake resistance, as measured by their β indices, when fed as body feed on the same weight basis or at the same level of C_S/C_F. In other words, pound for pound equivalent filter aids will have the same unit cake resistance.

This basis for equivalence is exemplified in the grouping of "equivalent filter aids" as a result of the statistical analysis of the data obtained in this study described on pages 58-59. The filter aids within the same group in the list on page 58 have unit filter cake resistances as measured by β that are not significantly different from the other filter aids within the group. In other words, the statistical variation in the β indices measured between the different lots of the same material and the different materials did not indicate there was a statistical difference in their unit hydraulic resistance as measured by β. They are, therefore, equivalent filter aids.

2. Filter aids can be considered equivalent when they provide "equivalent performance" (same headloss versus time relationship in a filter

run) as indicated by equivalent <u>total</u> filter cake resistance (equal values of βC_F) <u>on a weight basis</u>. Here equivalence would be expressed as:

$$x_1 lbS_2 = x_2 lbS_3 = x_3 lbS_4 = x_4 lbJ_4 \cdots\cdots x_n lbGl.$$

On this basis, the weight of each filter aid required to provide the same filter operating performance can be determined. For example, suppose that it is necessary to operate a precoat filter to remove 4 mg/l of iron at a filtration rate of 1 gpm/ft^2 using flat septa when the water temperature is 68°F and produce a filter run of 6 hrs at a terminal head-loss of 100 ft of water. If β-prediction equations are available which represent the performance of each of the filter aids when used for the filtration of the iron water, it is possible to use Eq. (7) in Table 3 to determine how many pounds of each filter aid body feed must be used per million gallons of water filtered to produce the desired filter performance. Then, it will be possible to compare filter aids on the basis of the <u>weights of each</u> required to provide equivalent performance. A comparison of the filter aids evaluated in this study on this basis will be made later in this section of the report.

3. Filter aids can be considered equivalent when they require the <u>same total volume</u> of filter aid to provide "equivalent performance" as indicated by equivalent total filter cake resistance (equal values of βC_F). Here, equivalence would be expressed as follows:

$$x_1 ft^3 S_2 = x_2 ft^3 S_3 = x_3 ft^3 S_4 = x_4 ft^3 J_4 \cdots x_n ft^3 Gl.$$

This basis for evaluating equivalence of filter aids is particularly significant in military operations. In the operation of standard water

precoat filtration equipment, filter aids are not fed either as precoat or as body feed on a weight basis. Instead, all operating procedures are, and will continue to be, based on the feeding of designated volumes of filter aid. Therefore, the third criteria for equivalent performance would be most meaningful in military specifications, since the military would like to know which filter aids are equivalent on a volume basis, regardless of their cost comparisons. Their choice of which filter aid to use, however, would be dictated by costs. A comparison of the filter aids evaluated in this study on a volume basis will be made later in this section.

4. Filter aids can be considered equivalent when the total cost of the filter aids required to provide "equivalent performance" is the same.

Since all filter aids do not cost the same amount on a per pound or a per cubic foot basis, the optimum economical approach would be to compare filter aid equivalence on the basis of the cost of the filter aid per million gallons required to provide equivalent performance. Comparison of the filter aids used in this study on this basis will be made later in this section.

General Observations

This study has shown that all of the filter aids tested produced water containing less than 0.06 mg/l of iron under identical filter operating conditions over the entire body feed range used (80 mg/l-500 mg/l range). This makes the choice of a filter aid easier in a practical situation than if some of the filter aids had not produced acceptable water.

The selection of a filter aid would be still more difficult if the range of body feed effective to produce quality water were different for the different filter aids. This study has revealed that all of the filter aids tested produced acceptable water quality, but with significantly different filter cake resistance.

This study has also demonstrated that β-prediction equations can be developed and are adequate to predict the filter cake resistance obtained when the filter aids are used to filter iron-bearing waters in which the solids are in the form of hydrated ferric oxide floc. The prediction equations can be used to predict filter cake resistance under any suspended solid and body feed concentration characteristic. This study has also substantiated earlier findings[18] that a prediction equation in the simplified form ($\beta = 10^{a} (C_S/C_F)^{b}$) is adequate for β prediction for such waters.

The question now arises as to what is a representative β-prediction equation for each grade and type of filter aid. In a practical situation, the engineer is confronted by the problem of selecting one filter aid in preference to any other. What he needs to do this, then, are the representative β-prediction equations (for the filtration of the water he plans to filter) for all filter aids to be considered. How these equations are used in the selection of a filter aid will be discussed later.

Representative β-Prediction Equations

The mean values of the exponents "a" and "b" of the β-prediction equations for each grade and type of filter aid were calculated (Appendix

I-F, Tables 22 and 23). The prediction equations using these mean values of the exponents for each filter aid were used for predicting values of β at designated levels of C_S/C_F. The values so calculated are listed in Appendix I-F, Table 24. These values of β were compared with the original range of values shown in Tables 9 through 11. There is close agreement between the two sets of values. The difference is generally much less than 10% with a large number of values within $\pm$ 3%. Therefore, it was concluded that β-prediction equations using such mean values for the exponents for each filter aid provide a β-prediction equation representative of that filter aid. These exponents are listed in Table 12.

Table 12. Summary of the representative β-prediction equations[a] for different filter aids for filtration of the synthetic water used in this study.

Filter aid	Values of exponents a =	b =
S2	10.275	2.121
S3	10.254	1.997
S4	10.347	2.037
J4	10.217	1.947
J3	10.247	1.956
J0	10.505	2.072
E6	10.524	2.127
E5	10.427	1.962
E2	10.535	2.009
G4	10.372	1.952
G1	10.345	1.859

[a] $\beta = 10^a (C_S/C_F)^b$.

Selection of Optimum Filter Aid

Now, it remains to demonstrate how these β-prediction equations can be used by an engineer to select the best filter aid in a given application.

Usually two types of situations are encountered in the field, one involving completely new designs and other situations where some of the plant components are already fixed. The former type of situation warrants the determination of the optimum combination of unit flow rate, body feed concentration, terminal headloss and the selection of a suitable filter aid that will result in production of filtered water at the least cost. Data required for such an analysis are: design flow in MGD, the β-prediction equations for the filter aids under consideration, filter aid cost per unit weight, first cost of the plant, its life and salvage value, precoat weight, power cost, labor cost, backwash cost, interest rate, precoat density and other details of the filtration plant such as septum diameter. The comparison of the optimum design combinations using the different types and grades of filter aid require repeated analyses. A computer program "POPO" has been developed at Iowa State University to simplify these analyses. Even in the absence of such a computer program, a few trial calculations can be made manually and an optimum combination of the design items can be determined. Manual analyses are laborious and less accurate. A filter aid which can be used to filter water with minimum optimum cost will be the most desirable. This study has provided β-prediction equations for various filter aids which are required for optimizing the design factors.

In other situations, some of the design factors may already be fixed such as the unit flow rate and the terminal head through the filters. The only factors that can then be optimized are the filter aid and the body feed concentration. The optimum combination of these two factors can also be determined in the same way to produce filtered water at the least cost. Still other situations exist where filter operating performance

has already been specified and it is desired to determine which filter aid will be the most economical to use. The β-prediction equations (Table 12) are useful in such cases as well. The following example illustrates this point.

For example, suppose it is desired to operate an existing precoat filter having flat septa for the filtration of an iron-bearing water similar to that used in this study. All of the filter aid grades evaluated in this study are to be considered for use. It is desired to filter a water containing 4 mg/l of iron (C_S = 4.0 mg/l) at a temperature of 68°F. The body feed rate (C_F) used with each filter aid must be such as to provide equivalent performance. The performance desired is such that the filter operated at a flow rate of 1 gpm/ft^2 will provide a filter run of 6 hrs when the pressure drop through the filter cake (H_c) is equal to 100 ft of water.

Equation (7) in Table 3 and the β-prediction equations in Table 12 can be used for this purpose. A summary of the performance criteria and sample calculations using both flat and cylindrical septa are included in Appendix I-G. Table 13 shows a summary of the body feed concentration and the amounts of the various filter aids required per million gallons of water on both a weight and a volume basis. The costs of the filter aids required are also shown in terms of dollars per million gallons of water filtered.

The results in Table 13 indicate that all of the filter aids could be used to remove 4 mg/l of iron from water at 60°F at a filtration rate of 1 gpm/ft^2 through flat septa and that all of them would produce 100 ft of headloss after 6 hrs of filtration. The weight of filter aid

Table 13. Amounts of the filter aids required for equivalent performance[a].

Filter aid designation	Unit cost[b] of filter aid, ¢/lb	Mean bulk[c] density, lb/ft^3	Body feed for equivalent performance[g]			
			C_F ppm	Cost, $/MG	Weight basis lb/MG	Volume basis ft^3/MG
S2	4.365	10.61	76	27.63[d]	633[e]	59.7[f]
S3	3.815	12.69	104	33.37	866	68.2
S4	3.715	13.01	112	34.66	933	71.7
J4	5.000	21.19	114	47.48	951	44.9
J3	4.900	19.56	118	48.16	983	50.3
J0	4.650	21.08	141	54.62	1174	55.7
E6	5.000	19.45	125	52.06	1041	53.5
E5	4.900	23.68	178	72.65	1483	62.6
E2	4.750	21.11	189	74.78	1574	74.6
G4	4.900	23.77	162	66.12	1349	56.8
G1	4.750	22.67	227	89.82	1891	83.4

[a]Removal of 4 mg/l of iron at 1 gpm/ft^2 at 68°F with a headloss (H_c) of 100 ft of water in a 6 hr run using flat septa.

[b]Approximate cost in carload lots delivered in Ames, Iowa.

[c]Mean values for a filter aid grade, Table 5.

[d] $\left(\frac{76 \text{ lb}}{10^6 \text{ lb}} \times 8.33 \frac{\text{lb}}{\text{gal}} \times 4.365 \frac{¢}{\text{lb}} \times \frac{\$}{100¢}\right) = \$27.63/10^6$ gal or \$27.63 \$/MG.

[e] $\left(\frac{76 \text{ lb}}{10^6 \text{ lb}} \times 8.33 \frac{\text{lb}}{\text{gal}}\right) = 633 \frac{\text{lb}}{10^6 \text{ gal}}$ or $633 \frac{\text{lb}}{\text{MG}}$.

[f] $633 \frac{\text{lb}}{\text{MG}} \times \frac{\text{ft}^3}{10.61 \text{ lb}} = 59.7 \frac{\text{ft}^3}{\text{MG}}$.

[g]On the basis of the multiple mean comparison tests, the variations in the filter aids in different production lots are such that there is no significant differences, on a C_F basis, in filter aids grouped as follows:

Group 1 S2
Group 2 S3, S4, J4, J3
Group 3 J0, E6
Group 4 E5, G4
Group 5 E2 and G1.

Thus, there is not a significant difference in C_F values of 104, 112, 114, and 118 for S3, S4, J4, and J3, respectively. Any given shipment of any of these materials might require a C_F between 104 and 118 for equivalent performance. Average amounts over a period of time would tend to give average amounts listed in this table.

required as body feed per million gallons of water would vary from 633 lbs using S2 to 1891 lbs using G1.

The cost of the filter aid at Ames, Iowa under an operating system where body feeds were fed on a weight basis would vary from $27.63 to $89.82 per million gallons. In view of the variation in the characteristics of each grade of material from shipment to shipment, a statistical analysis would indicate that the filter aids which are equivalent could be grouped as follows:

Group 1 S2 (633)*

Group 2 S3 (866), S4 (933), J4 (951), J3 (983)

Group 3 J0 (1174), E6 (1041)

Group 4 E5 (1483), G4 (1349)

Group 5 E2 (1574), G1 (1891).

Since the weight of each material required to provide the filter performance cited would vary from shipment to shipment (the figures in Table 13 are average values characteristic of the filter aids), any of the materials within a group can be considered equivalent. In other words, any given lot of the filter aids within the same group may require the same weight of filter aid as body feed per million gallons as any other given lots of the other filter aids to provide the same performance. In this case, if Group 2 filter aids were to be used, it is possible that different lots of S3, S4, and J3 would all have to be fed in amounts of 951 pounds per million gallons to provide the same performance. On the long term average, however, the body feed requirements in Table 13 would be approached. At the prices listed in Table 13, the Group 2 perlite filter aids would provide lower filtration costs than the Group 2 diatomite filter aids.

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*Weight of filter aid in pounds required as body feed per million gallons.

In a military situation, filter aids are fed both as precoat and as body feed on a volume basis. Thus, in a military situation, the emphasis is in establishing a new grouping of filter aids in which the materials may be expected to provide the same equivalent performance when fed as body feed in the same volume amounts. In military operations, the filter aids commonly used have included J4, J3, E6 and G4 with approximately equivalent performance. The data in Table 13 can be rearranged in groups requiring approximately equal volumes of body feed to provide equal performance as follows

Group 1	J4 (44.9)*
Group 2	J3 (50.3), E6 (53.5), J0 (55.7), G4 (56.8), S2 (59.7)
Group 3	E5 (62.6), S3 (68.2)
Group 4	S2 (71.7), E2 (74.6), G1 (83.4)

In military operations, the use as body feed of 55 ft^3 of filter aid per million gallons would produce runs of varying lengths. In general, E6, J0, and G4 filter aids would give essentially the same run length (6 hrs at 100 ft of head). The use of J3 in that amount would provide a longer filter run. The use of S2 in that amount would produce a shorter filter run. Here, the use of J4 would provide the longest filter run. On an equal volume feeding basis, the diatomite filter aids provide better performance than the perlite filter aids.

Using the data in Table 13, it is possible to demonstrate the economy involved in using the different filter aids on a volume basis. Assume that a

*Body feed required in ft^3/MG.

run of 6 hrs is desired under the filter operating conditions of Table 13 and it has been decided to feed 55 ft^3 of filter aid per million gallons as body feed. Therefore, it is necessary to determine the cost (at Ames, Iowa) of operating the filters with Group 1 and 2 filter aids. The costs using each material would be approximately as follows.

Filter aid	Cost, $/MG	Effect
J4: $(\frac{55}{44.9} \times 47.48)$	$58.16	Run length increased significantly
J3: $(\frac{55}{50.3} \times 48.16)$	$52.66	Run length increased somewhat
E6: $(\frac{55}{53.5} \times 52.06)$	$53.52	Run length desired
J0: $(\frac{55}{55.7} \times 54.62)$	$53.93	Run length desired
G4: $(\frac{55}{56.8} \times 66.12)$	$64.02	Run length shortened somewhat
S2: $(\frac{55}{59.7} \times 27.63)$	$25.46	Run length shortened significantly

Using E6 at a body feed rate of 55 ft^3/MG instead of the rate of 53.5 ft^3/MG required to give a 6 hr run would give a run length slightly longer than 6 hrs. Using J4 at a body feed rate of 55 ft^3/MG instead of the rate of 44.9 ft^3/MG required to give a 6 hr run would give a run significantly longer than 6 hrs and some of the filter aid (55.0-44.9 or 10.1 ft^3/MG) would be wasted. Thus, the total cost of the filter aid increases to the values shown above. In this case, economics indicate that the J3 filter aid provides the lowest operating cost ($52.66/MG) of all of the filter aids which provide a run of 6 hrs or longer. However, the use of S2 must be considered also, since its cost is significantly under the cost of the other filter aids ($25.46/MG for S2 as compared with $52.66/MG for J3). However, any decision to use S2 must be tempered by the knowledge that the

run length would be less than 6 hrs and might negate its suitability. Equation (7) can be used to evaluate the length of run that would result if it were used at a body feed rate of 55 ft^3/MG.

The data in Table 13 were developed on the basis of using flat septa. Many precoat filters, including military filters, make use of cylindrical septums. The "increasing area effect" inherent in the use of cylindrical septa provides a larger surface area as the filter cake builds up. Both the flow rate per unit cake area and the cake thickness per unit of water filtered decrease with time. As a result, under identical filter operating conditions, cylindrical septa provide longer filter runs than flat septa. The smaller the diameter of the cylindrical septa the more pronounced is the increasing area effect.

As compared with use of flat septa, the increasing area effect using cylindrical septa can be used either to provide longer filter runs under the same operating conditions or to provide the same length of run by reducing the amount of body feed used. Equation (5) in Table 3 can be used to determine the body feed requirements using cylindrical septa for the filter operating conditions listed in Table 13. Sample manual calculations are included in Appendix I-G. The body feed required with flat and 3½ in. O.D. and 1 in. O.D. cylindrical septa to provide a filter run of 6 hrs with a terminal headloss of 100 ft of water when removing 4 mg/l of iron from water at 68°F at a filtration rate of 1 gpm/ft^2 of septa area are tabulated below.

Filter aid	Septa	C_F/mg/l	Body feed for equivalent performance		
			lb/MG	$/MG	ft^3/MG
S2	Flat	76	633	$27.63	59.7
	3½ in. O.D.	59	491	$21.43	44.8
	1 in. O.D.	40	333	$14.54	31.4
J3	Flat	118	983	$48.16	50.3
	3½ in. O.D.	96	800	$39.20	41.0
	1 in. O.D.	70	583	$28.57	29.8

These results effectively demonstrate the significantly beneficial effect of cylindrical septa over flat septa in reducing body feed requirements required to produce a given filter operating result.

Another important factor may also have to be considered in selecting one filter aid in preference to others. A filter aid which has less bulk density will produce a thicker cake for the same weight of material and may make longer filtration runs impractical in the plants with septa too close together. In new plants the increased cost of a filter with a bigger housing due to the increased space needed between the septa may have to be weighed against the expected savings by the use of a filter aid which has a bulk density requiring added filter cake thickness. It is possible to calculate the filter cake thickness that will build up in a certain run length at a desired body feed concentration. This should be checked so that the maximum cake thickness that builds up will permit adequate space between the adjacent septa. This limitation may dictate the use of some particular filter aid in a plant with septa that do not provide adequate space for filter cake buildup.

SUMMARY AND CONCLUSIONS

The results of this study are summarized below:

1. All the filter aids tested in this study produced an acceptable filtered water containing less than 0.06 mg/l of iron over the entire range of body feed which was about 50-500 mg/l.
2. The simplified β-prediction equations ($\beta = 10^{a} (C_S/C_F)^{b}$) was found adequate to predict closely the filter cake resistance using iron bearing water (Fe^{+++}).
3. The representative β-prediction equations (Table 12) can be used to predict filter cake resistance when the various filter aids are used to remove iron from a water simulating military filtration experience.
4. Under identical conditions, filter aid S2 developed the least filter cake resistance as measured by the β-index as compared to the other filter aids on an equivalent weight basis. Filter aid G1 similarly produced the highest filter cake resistance. Other filter aids produced cake resistance falling in a middle range.
5. The diatomite filter aids produced lower values of filter cake resistance as measured by the β-index than the perlite filter aids when used on an equivalent volume basis (page 53).
6. Statistical analyses declared certain filter aids not significantly different on an equivalent weight basis from others on the basis of results available. These groups have been listed on page 58 in an increasing order of filter cake resistance.
7. On an equivalent performance basis (Table 13) the perlite filter aids required less weight of filter aid but larger volumes of filter aid than would be required with some diatomite filter aids.

8. In military operations where body feeding is and necessarily must be accomplished on a volume basis, less volume is required of several diatomite filter aids (J4, J3, E6, J0, and G4) than of perlite filter aids to obtain equivalent performance.

9. The selection of one filter aid fo. use in a given application in preference to any other is a matter of cost comparison. A filter aid which requires the least weight or volume to provide the required performance characteristics may not be the most economical to use if its cost on a weight or volume basis is out-of-line with the other materials. The cost of the filter aids at the point of use would dictate the choice from time to time. This study has provided sufficient information to optimize the least cost combination of various design factors such as unit flow rate, body feed concentration, terminal headloss, and type and grade of the filter aid. Under situations where some of the design factors are already fixed, the results of this study are equally useful to determine an optimum combination of the other factors.

10. The use of cylindrical septa will reduce significantly the body feed required to obtain equivalent performance as compared to the use of flat septa.

11. In filter plants in which the distance between septa is limited, the use of filter aids having low bulk density may impose limitations as to run length. This may have to be considered when selecting a particular filter aid for use. It is possible to calculate filter cake thickness for various body feed concentrations knowing the bulk density of the filter aid.

RECOMMENDATIONS

The results of this study were obtained by using 11 different filter aids (8 diatomites and 3 perlites) to filter an iron-bearing water simulating the effluent from a military water pretreatment unit called an Erdlator. It is very dangerous to assume that the results reported in this study could be extrapolated for use in the removal of other suspended solids in other situations. Here, all of the filter aids removed the suspended solid, iron, from the water to produce the desired quality of effluent. Thus, the comparison of the filter aids was made on the basis of the weights and volumes of each required to provide equivalent performance.

With other suspended solids, the differences between the filter aids in their physical and hydraulic characteristics may produce significantly different performance. Therefore, the authors strongly recommend that the techniques described in this report be used to evaluate quantitatively the new situation which would result when other suspended solids are to be removed by filtration under other conditions. Extrapolation of the results of this study to cover filtration of other suspended solids without such quantitative verification is not warranted.

Additional filter aids of each grade and type should be tested in the laboratory so that the representative β-prediction equations are more accurate. This is particularly true for the filter aids which had a sample size of only 3. A minimum of 5 to 6 lots of each filter aid would narrow the 95% confidence limits on β-prediction using the various filter aids. The β-prediction equations (Table 12) could then be modified accordingly.

Additional tests should be made using suspended solids that actually discharge from an Erdlator under field operating conditions. Such tests would serve to confirm whether the laboratory test conditions used are, in fact, representative of military field conditions.

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APPENDIX I-A

Determination of In-Place Bulk Density*

General

One of the primary factors that affects the flow resistance of filter cakes produced in any filtration process is the thickness of the filter cake which develops. This in turn, depends on the in-place bulk density of the cake. The in-place bulk density of the cake, which equals the weight per unit of in-place volume, can be obtained from the dry weight of the cake solids and their in-place volume.

Preliminary investigations were made to determine the effect of pressure drop across the filter cake on in-place bulk density measurements. The tests results indicated that the in-place bulk density was relatively independent of the applied pressure, if the pressure drop across the filter cake was greater than 5 psi. Filtration under pressures greater than 20 psi, however, produced smoother filter cakes. The settling of large particles during a test to determine bulk densities of filter aids, which was present at low pressure drops, was nearly eliminated at high pressure differentials. In view of these observations, a pressure drop of 30 psi was used in making all of the in-place bulk density tests.

Apparatus

Figure 11 shows a schematic diagram of the laboratory filtration apparatus used for the determination of the in-place bulk densities of

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*A "Standard Procedure for the Determination of Bulk Density and Filter Cake Resistance of Precoat Filter Aids," developed later in this study, is included as Part IV of the Final Report.

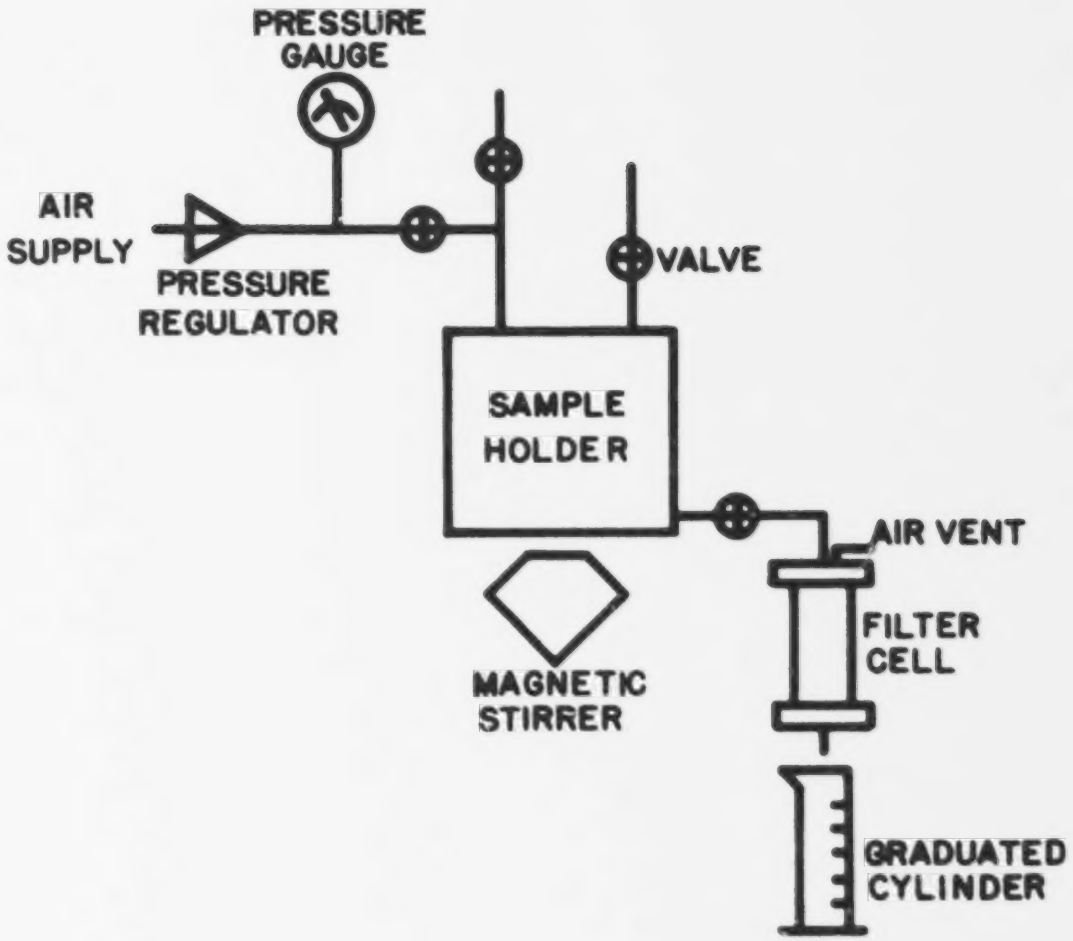

Fig. 11. Schematic diagram of the in-place bulk density determination apparatus.

the filter aids. It essentially consists of a sample holder to house the water sample to be filtered, a lucite filter cell with a filter medium consisting of a millipore filter paper and its support. The system was connected to an air supply line and the air pressure was controlled by means of a pressure regulator. The filtrate was collected in a graduated cylinder placed near the outlet end of the filter cell.

Millipore filter papers of 37 mm diameter and 0.45 μ pore size were used to filter the filter aid out of the suspension.

Procedure

A filter aid suspension was prepared by adding a small quantity of the filter aid (about 5-10 g) to a standard amount of distilled water (about 1 liter). It is advisable to use approximately the same amount of the filter aid and water every time, so that neither too thick nor too thin a cake is formed. The resulting suspension was left for about half an hour, with occasional stirring, to enable it to reach chemical equilibrium.

At the beginning of any filtration experiment, the pressure regulator was set to the desired pressure of 30 psi. The filter aid suspension was introduced to the sample holder and kept under continuous mixing using a magnetic stirrer. The filter cell containing a new millipore filter paper was then connected to the system. The water suspension, which was under atmospheric pressure in the sample holder, was allowed to flow to the filter cell until all the air was expelled.

The filtration process was then started. Thus, the suspension was forced to flow through the filter cell producing a filter cake at the surface of the filter paper. The filtrate was collected in the graduated cylinder below. The process was continued until all the suspension was filtered out. The filter cell was disassembled and the cake thickness measured as accurately as possible. The cake was pushed out of the cell, dried in a 103°C oven overnight, and weighed.

The in-place bulk density was calculated by dividing the dry weight of the cake by the in-place volume of the cake.

APPENDIX I-B

Example Illustrating Manual

Computation of S-Index

Data from Run 57, Filter Aid S2-1

Headloss in inches of Hg	Minutes elapsed since start of run
0.4	0
3.6	15
7.0	30
10.7	45
13.8	60
16.7	75
19.7	90
22.7	105
25.3	120
28.2	135
30.65	150

C_S = 2.08 mg/l

C_F = 301.1 mg/l

Temperature = 60°F

Bulk density = 11.31 lbs/ft^3

Diameter of septum = 3.5 in.

Precoat = 0.15 lb/ft^2

Dilution factor = 18/hr

Calculations: (See Table 3 for equations and meaning of symbols.)

Find R_S:

$$R_S = \frac{3.5}{2 \times 12} = 0.1458 \text{ ft}$$

Find R_o:

$$R_o = R_S + L_p, \text{ where } L_p = \frac{w}{\gamma_p}$$

$w = 0.15$ lb/sq ft

$\gamma_p = 11.31$ lbs/cu ft

$$\therefore \quad L_p = \frac{0.15}{11.31} \text{ lb/ft}^2 \times \text{ft}^3/\text{lbs}$$

$$= 0.0133 \text{ ft}$$

$$\therefore \quad R_o = R_S + L_p$$

$$= 0.1458 + 0.0133 \text{ ft}$$

$$= 0.1591 \text{ ft}$$

Find ϕ:

$$\phi = 2q\gamma_w C_D (10^{-6})/\gamma_p$$

$$q = 1 \text{ gpm/ft}^2$$

$$= 1.0 \text{ gal/min ft}^2 \times 60 \text{ min}/1.0 \text{ hr} \times 1.0 \text{ ft}^3/7.48 \text{ gal}$$

$$= 8.0214 \text{ ft/hr}$$

$$\gamma_w = 62.4 \text{ lbs/ft}^3$$

$$C_D = 301.1 \text{ ppm}$$

$$\gamma_p = 11.31 \text{ lbs/ft}^3$$

$$\therefore \phi = 2q\gamma_w C_D (10^{-6})/\gamma_p$$

$$\phi = 2 \times 8.0214 \text{ ft/hr} \times 62.4 \text{ lbs/ft}^3 \times 301.1 \times 10^{-6}/11.31 \text{ lbs/ft}^3$$

$$\phi = 28.191 \times 10^{-3} \text{ ft/hr}$$

Plot: H versus $\text{Ln}(1 + R_S \phi x/R_o^2)$ as shown in Fig. 12. This gives a straight line with slope $R_S\sigma/\phi$ of 95 ft. The term within the bracket and its natural logarithm will have to be calculated (Table 14).

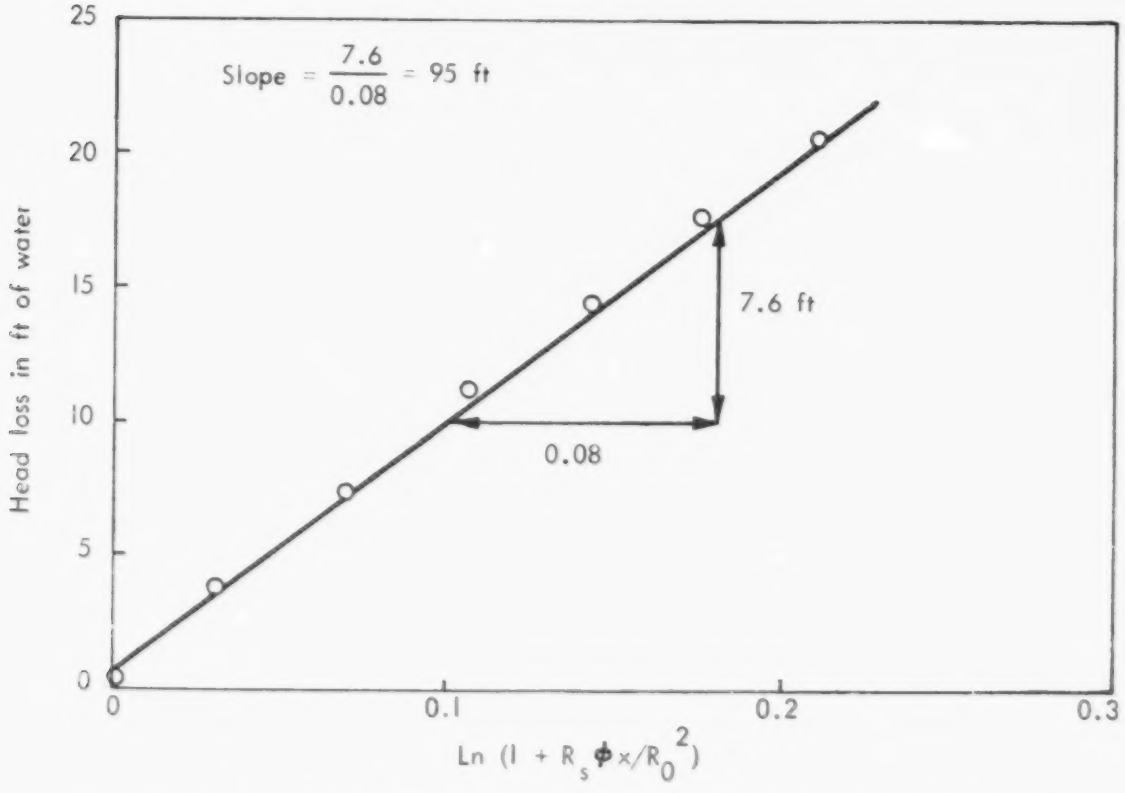

Fig. 12. Plot of headloss versus $\text{Ln}(1 + R_S \phi x/R_o^2)$ for data obtained in run 57.

Since, Slope = $R_S\sigma/\phi$

or 95 ft = 0.1458 ft $\times$ $\sigma/28.191 \times 10^{-3}$ ft/hr

$\therefore$ $\sigma = 95 \times 28.191 \times 10^{-3}/0.1458$ ft/hr

$= 18.369$ ft/hr.

Table 14. Computations of $\text{Ln}\left(1 + \frac{R_S^2 \delta X}{R_o^2}\right)$

Headloss In. of Hg	Headloss Ft of water	Time t Min	Time t Hrs	$X = t - \frac{1 - e^{-\delta t}}{\delta}$	$1 + \frac{R_S^2 \delta X}{R_o^2}$	$\text{Ln}\left(1 + \frac{R_S^2 \delta X}{R_o^2}\right)$
0.4	0.420	0	0.00	0.0000	1.00000	0.00000
3.6	3.780	15	0.25	0.1951	1.03168	0.03117
7.0	7.350	30	0.50	0.4445	1.07218	0.06952
10.7	11.235	45	0.75	0.6944	1.1128	0.10660
13.8	14.490	60	1.00	0.9444	1.1534	0.14265
16.7	17.535	75	1.25	1.1944	1.1940	0.17729
19.7	20.685	90	1.50	1.4444	1.2345	0.21106
22.7	23.835	105	1.75	1.6999	1.2760	0.24372
25.30	26.565	120	2.00	1.9444	1.3157	0.27353
28.20	29.610	135	2.25	2.1944	1.3563	0.30475
30.65	32.1825	150	2.50	2.4444	1.3969	0.33424

Find β:

Since $\sigma = q^2 \nu \beta C_D / g$

$\therefore \beta = g\sigma / q^2 \nu C_D$

where

$q = 8.0214$ ft/hr

$\nu = 1.217 \times 10^{-5}$ ft^2/sec

$= 1.217 \times 10^{-5} \times 3600$ ft^2/hr

$= 4.3812 \times 10^{-2}$ ft^2/hr

C_F = 301.1 ppm

$g = 32.2 \text{ ft/sec}^2$

$= 32.2 \times 3600 \times \text{ft/hr}^2$

$= 417.31 \times 10^6 \text{ ft/hr}^2$

$\beta = g\sigma/q^2 \nu C_D$

$$= \frac{417.31 \times 10^6 \text{ ft/hr}^2 \times 18.369 \text{ ft/hr}}{8.0214 \times 8.0214 \text{ ft}^2/\text{hr}^2 \times 4.3812 \times 10^{-2} \text{ ft}^2/\text{hr} \times 301.1}$$

$= 9.031 \times 10^6/\text{ft}^2$.

The value of β obtained using the "BID" computer program was 8.905 $\times 10^6/\text{ft}^2$. The percentage difference in β between manual and computer fitting of the slope of the line in Fig. 12 was:

$$\text{Percentage error} = \frac{9.031 - 8.905}{8.905} \times 100$$

$$= \frac{0.126}{8.905} \times 100$$

$= 1.42\%$.

APPENDIX I-C

Table 15. Summary of results of filtration runs.

Run No.	Filter aid	C_s in mg/ℓ	C_p in mg/ℓ	C_p/C_s	Multiple correlation coefficient, R, in percent	Filter cake resistance, β in ft-2/lb	Run No.	Filter aid	C_s in mg/ℓ	C_p in mg/ℓ	C_p/C_s	Multiple correlation coefficient, R, in percent	Filter cake resistance, β in ft-2/lb
59	S2-4	7.54	1.60	0.0644	99.910	25.64	197	ES-3	7.8	229.8	0.0144	99.611	34.00
60	S2-4	7.53	189.0	0.0391	99.849	12.50	198	ES-3	7.72	165.8	0.0665	99.673	65.65
61	S2-4	7.62	282.4	0.0390	99.957	10.47	199	ES-3	7.78	134.5	0.0233	99.711	15.11
62	S2-4	7.17	109.4	0.0232	99.916	7.287	200	ES-3	7.79	66.2	0.0168	99.821	8.761
63	S2-4	7.46	108.0	0.0242	99.925	7.251							
64	S2-4	7.37	51.15	0.1442	99.003	36.7	192	ES-4	7.97	160.7	0.0234	99.771	17.87
65	S2-4	7.24	69.50	0.1058	99.594	18.9	193	ES-4	7.94	109.5	0.0171	99.818	9.830
							194	ES-4	7.71	58.6	0.0161	99.911	6.828
66	S2-5	7.60	49.42	0.1149	98.567	216.0	195	ES-4	7.83	219.1	0.0317	99.918	41.82
67	S2-5	6.69	70.74	0.1419	97.904	145.8	196	ES-4	7.53	153.9	0.0289	99.807	82.21
68	S2-5	7.26	117.6	0.0819	99.499	58.49							
69	S2-5	9.52	166.2	0.0574	99.817	38.27	205	ES-1	7.56	134.8	0.0351	99.710	40.11
70	S2-5	8.06	140.6	0.0625	99.980	22.05	206	ES-1	7.40	103.5	0.0442	99.682	66.62
71	S2-5	7.91	237.4	0.0308	99.915	13.63	207	ES-1	7.60	82.93	0.0252	99.834	18.47
72	S2-5	8.20	242.1	0.0281	99.972	10.15	208	ES-1	7.37	44.2	0.0166	99.821	9.169
73	S2-1	6.95	140.8	0.0432	99.602	50.66	220	ES-4	8.06	138.4	0.0311	99.903	17.29
74	S2-1	7.21	155.1	0.0464	99.871	27.59	221	ES-3	8.12	89.6	0.0313	99.894	10.25
75	S2-1	7.41	280.5	0.0304	99.962	18.58	222	ES-3	7.85	84.2	0.0214	99.944	6.491
76	S2-1	8.02	207.9	0.0386	99.876	17.28	223	ES-3	7.97	131.6	0.0330	99.726	13.78
77	S2-1	8.08	301.1	0.0268	99.980	8.905	224	ES-3	7.99	71.4	0.0158	99.708	45.28
78	S2-1	7.86	111.8	0.0691	99.847	91.58							
80	S2-1	7.87	301.4	0.0275	99.927	87.96	230	ES-2	.72	134.5	0.0360	99.731	107.227
							231	ES-2	.73	115.8	0.0412	99.786	67.4288
81	S2-20	7.79	92.8	0.0839	99.841	98.20	232	ES-2	7.68	95.2	0.0295	99.877	28.9015
82	S2-20	7.96	78.4	0.1012	99.842	165.9	234	ES-2	7.80	89.2	0.0202	99.909	12.4812
83	S2-20	7.84	192.7	0.0406	99.862	65.76	235	ES-2	7.93	52.4	0.0151	99.880	7.68088
84	S2-20	7.99	327.1	0.0608	99.916	52.08							
85	S2-20	7.93	148.6	0.0534	99.915	35.76	142	GA-1	7.82	248.5	0.0296	99.863	27.66
86	S2-20	7.91	252.0	0.0314	99.966	13.11	143	GA-1	7.90	140.5	0.0421	99.830	55.41
							144	GA-1	7.76	98.4	0.0251	99.607	18.67
87	S2-5	8.08	121.0	0.0615	99.815	45.69	145	GA-1	7.77	67.7	0.0182	99.865	11.21
123	S2-5	8.52	162.5	0.0501	99.998	29.10	146	GA-1	7.73	40.8	0.0165	99.850	9.125
124	S2-5	7.85	70.5	0.1112	99.264	191.0							
125	S2-5	8.08	233.4	0.0412	99.978	17.90	209	GA-2	7.5	119.1	0.0222	99.662	12.98
126	S2-5	7.88	291.0	0.0268	99.951	8.780	210	GA-2	7.59	45.4	0.0165	99.888	7.230
							212	GA-2	8.00	226.3	0.0425	99.736	29.96
127	S2-9	7.72	128.2	0.0600	99.897	47.12	213	GA-2	7.90	119.6	0.0468	99.448	53.87
128	S2-9	8.06	89.5	0.0902	99.870	125.0							
129	S2-9	8.10	152.2	0.0545	99.976	37.72	226	GA-3	8.05	24.5	0.0135	99.885	7.98
130	S2-9	8.21	280.5	0.0509	99.990	22.23	227	GA-3	7.74	184.1	0.0419	99.657	46.31
131	S2-9	7.91	265.7	0.0297	99.982	11.89	228	GA-3	7.89	330.0	0.0239	99.473	15.20
							229	GA-3	7.43	453.0	0.0175	99.212	8.592
142	S2-6	8.11	258.6	0.0313	99.994	11.71							
143	S2-6	7.87	88.1	0.0892	99.810	106.8	231	GA-4	7.85	68.1	0.0830	99.558	13.14
144	S2-6	7.81	195.7	0.0401	99.919	17.34	232	GA-4	8.18	214.0	0.0263	99.798	7.658
145	S2-6	7.56	151.8	0.0499	99.892	26.52	233	GA-4	7.99	65.3	0.0127	99.909	5.061
146	S2-6	7.88	311.85	0.0252	99.971	8.041	234	GA-4	8.00	111.2	0.0479	99.681	48.89
							235	GA-4	7.77	37.5	0.0126	99.785	27.86
169	S2-21	7.70	112.70	0.0686	99.971	68.79							
170	S2-21	8.03	158.8	0.0503	99.968	30.07	236	GA-5	8.02	114.5	0.0459	99.803	53.76
171	S2-21	8.02	215.3	0.0372	99.968	18.22	237	GA-5	8.00	46.3	0.0166	99.777	27.86
172	S2-21	8.07	149.5	0.0537	99.989	14.55	238	GA-5	7.91	135.6	0.0451	99.784	13.81
173	S2-21	8.13	296.9	0.0275	99.959	10.81	239	GA-5	7.77	61.0	0.0156	99.841	8.162
							240	GA-5	.94	55.0	0.0147	99.856	5.433
28	S1-4	7.50	124.0	0.0605	99.784	54.51							
60	S1-4	7.21	125.6	0.0575	99.856	51.05							
61	S1-4	7.07	138.8	0.0495	99.901	59.06	147	GA-1	7.45	229.8	0.0325	99.901	35.50
62	S1-4	7.31	96.5	0.0737	99.890	107.3	148	GA-1	8.17	95.9	0.0178	99.949	25.33
63	S1-4	7.21	68.9	0.1055	99.681	211.1	149	GA-1	7.76	172.5	0.0308	99.991	15.35
64	S1-4	7.09	247.9	0.0286	99.916	14.29	150	GA-1	7.66	479.1	0.0159	99.935	11.24
65	S1-4	7.36	180.6	0.0407	99.736	27.06	151	GA-1	7.66	163.6	0.0468	99.893	73.58
66	S1-4	8.00	311.4	0.0257	99.920	11.34							
67	S1-4	8.47	141.6	0.0598	99.932	10.04	214	GA-2	7.68	190.2	0.0333	99.908	47.45
1	S1-4	7.62	412.1	0.0184	99.951	6.508	215	GA-2	7.77	161.8	0.0475	99.890	86.21
							216	GA-2	7.46	98.6	0.0242	99.946	23.23
							217	GA-2	7.43	57.4	0.0177	99.950	11.66
							218	GA-2	7.75	411.00	0.0212	99.946	23.85
							219	GA-2	7.75	562.7	0.0118	99.961	9.158

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1	S1-2	8.06	111.0	0.0276	99.857	96.44
2	S1-2	8.27	165.8	0.0198	99.931	42.57
25	S1-2	7.77	207.9	0.0143	99.862	24.88
26	S1-2	7.55	296.2	0.0251	99.977	13.91
27	S1-2	7.51	339.9	0.0221	99.950	8.420
28	S1-2	7.51	165.8	0.0456	99.809	15.96
29	S1-1	8.09	153.5	0.0527	99.778	49.22
30	S1-1	7.75	148.9	0.0287	99.810	38.59
31	S1-1	7.75	207.1	0.0196	99.866	26.52
32	S1-1	8.20	253.3	0.0332	99.819	17.27
33	S1-1	7.56	299.1	0.0252	99.978	10.53
34	S1-1	7.62	86.0	0.0208	99.987	9.065
35	S1-1	7.56	99.0	0.0765	99.938	117.8
36	S1-1	7.41	106.0	0.0698	99.818	107.0
37	S1-4	8.96	111.8	0.0624	98.824	78.76
38	S1-4	7.78	161.0	0.0484	99.898	62.60
39	S1-4	7.85	226.1	0.0347	99.975	23.61
40	S1-4	7.62	299.2	0.0306	99.971	17.22
41	S1-4	7.59	339.3	0.0241	99.900	11.42
42	S1-4	7.59	189.4	0.0195	99.973	8.792
43	S1-3	8.06	111.4	0.0685	99.949	59.24
44	S1-3	7.91	116.5	0.0660	99.928	89.09
45	S1-3	7.98	81.7	0.0956	99.756	196.1
46	S1-3	8.21	204.2	0.0432	99.866	26.76
47	S1-3	8.15	278.1	0.0293	99.922	13.86
48	S1-2	7.78	132.8	0.0586	99.896	78.95
49	S1-2	7.73	87.03	0.0890	99.684	182.5
50	S1-2	7.88	136.1	0.0555	99.944	50.28
51	S1-2	8.00	217.8	0.0368	99.919	24.06
52	S1-2	8.08	301.1	0.0271	99.957	14.38
53	S1-2	8.03	422.4	0.0190	99.986	9.339
54	S1-1	8.23	125.8	0.0665	99.886	101.6
55	S1-1	8.17	165.0	0.0445	99.959	38.16
56	S1-1	8.17	247.0	0.0331	99.933	21.79
57	S1-1	8.24	304.1	0.0268	99.969	15.31
58	S1-1	8.27	414.0	0.0199	99.992	9.926
59	E5-2	7.86	212.2	0.0338	99.581	23.96
60	E5-2	7.94	171.6	0.0462	99.531	46.60
61	E5-2	7.99	136.1	0.0580	99.741	76.79
62	E5-2	8.01	303.3	0.0214	99.880	8.915
63	E5-2	7.86	308.1	0.0235	99.769	13.01
64	E5-2	8.17	436.0	0.0187	99.772	6.866
65	E5-2	7.98	511.3	0.0150	99.916	4.845
66	E5-2	7.62	167.1	0.0457	99.695	48.48
67	E5-1	8.23	180.0	0.0278	99.768	12.97
68	E5-1	8.14	496.4	0.0186	99.825	6.251
69	E5-1	8.10	548.2	0.0158	99.868	4.888
70	E5-1	8.02	234.3	0.0352	99.724	29.14
71	E5-1	7.78	312.3	0.0236	99.686	20.57
72	E5-1	7.83	422.4	0.0186	99.511	12.69
73	E5-1	7.73	601.8	0.0128	99.625	6.161
74	E5-1	8.10	535.8	0.0151	99.685	8.710
75	E5-17	8.04	346.1	0.0232	99.614	13.91
76	E5-17	8.16	460.8	0.0177	99.851	8.533
77	E5-17	8.04	538.7	0.0149	99.910	6.355
78	E5-17	7.78	253.4	0.0307	99.846	25.66

251	G1-5	7.57	154.7	0.0490	99.935	88.05
252	G1-5	8.27	197.2	0.0320	99.935	52.28
253	G1-5	8.42	253.0	0.0260	99.666	21.79
254	G1-5	8.02	411.0	0.0195	99.806	12.69
255	G1-5	8.32	485.0	0.0172	99.918	10.12
107	14-6	8.3	121.3	0.0429	99.688	19.83
108	14-6	8.27	248.7	0.0242	99.766	20.13
109	14-6	8.32	332.1	0.0252	99.679	11.59
110	14-6	8.06	417.9	0.0193	99.836	8.594
152	14-4	8.59	214.1	0.0410	99.868	30.15
153	14-4	7.63	157.6	0.0685	99.610	52.15
154	14-4	7.94	264.4	0.0299	99.808	16.67
155	14-4	8.04	345.7	0.0232	99.853	10.36
156	14-4	7.72	429.1	0.0182	99.865	6.668
260	14-7	7.89	169.1	0.0466	99.789	50.81
261	14-7	7.82	35.5	0.2200	98.710	701.969
262	14-	7.83	249.1	0.0185	99.891	2.54402
263	14-7	7.79	1999.7	0.0071	99.916	1.11916
264	14-7	7.40	1955.25	3.0010	99.810	0.5465-2
93	13-5	8.00	233.1	0.0362	99.964	22.50
94	13-5	7.65	328.4	0.0232	99.909	11.01
95	13-5	7.72	381.6	0.0202	99.956	8.349
96	13-5	7.67	191.6	0.0400	99.781	29.74
97	13-5	7.49	116.1	0.0656	99.736	79.53
98	13-6	7.55	201.0	0.0372	99.540	28.32
99	13-6	7.17	128.7	0.0556	99.684	65.61
100	13-6	7.26	214.1	0.0340	99.685	22.21
101	13-6	7.89	250.8	0.0314	99.969	18.08
102	13-6	8.23	352.0	0.0235	99.972	10.17
163	13-7	7.80	201.0	0.0384	99.681	34.75
165	13-7	7.93	277.6	0.0286	99.851	17.56
166	13-7	8.01	399.1	0.0217	99.847	11.50
167	13-7	7.79	561.2	0.0169	99.945	6.612
168	13-7	7.65	551.9	0.0138	99.969	4.589
103	10-5	8.32	207.3	0.0414	99.065	41.43
10	10-5	8.26	294.1	0.0298	99.813	20.27
10	10-5	8.28	349.0	0.0241	99.896	12.78
10	10-5	8.26	486.5	0.0442	99.515	49.46
1	10-1	8.08	296.0	0.0273	99.889	20.30
1	10-1	8.11	228.9	0.0354	99.763	32.66
161	10-1	8.25	328.0	0.0258	99.881	12.34
162	10-1	8.27	556.2	0.0182	99.916	9.219
163	10-1	8.22	170.8	0.0484	99.820	56.52
265	10-6	7.91	261.9	0.0315	99.825	21.41
266	10-6	7.60	195.6	0.0192	99.905	9.571
267	10-6	7.63	458.7	0.0167	99.917	8.571
268	10-6	7.52	110.6	0.0685	99.648	148.3
269	10-6	7.67	137.8	0.0557	99.454	76.26

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APPENDIX I-D

Comparison of Sil-Flo Perlites and Diatomaceous Earths[a] (On Basis of Equivalent Volumes)

Sil-Flo filter aids		Diatomaceous earth filter aids			
Grade nos.	Approx. wet density lb/cu ft	Eagle-Picher Celatom grades	Johns-Manville Celite grades	Dicalite grades	Approx. wet density lb/cu ft
190	10.1	FW-80	560	5000	20.0
SPG	10.3	Celatom SP	Celite SP	Dicalite SP	18.9
272	10.5	FW-60	545	4200	18.9
332	12.4	FW-20	503	Speedex	18.4
443	13.0	FW-14 H	Hyflo Super-Cel	Speedplus	18.5
634	14.0	FW-6	512	Special Speedflo	19.5
734	15.5	FP-4	Standard Super-Cel	Speedflo	20.22

[a]Source: "Technical Data," Bull. No. 10, Sil-Flo Corp., Fort Worth, Texas.

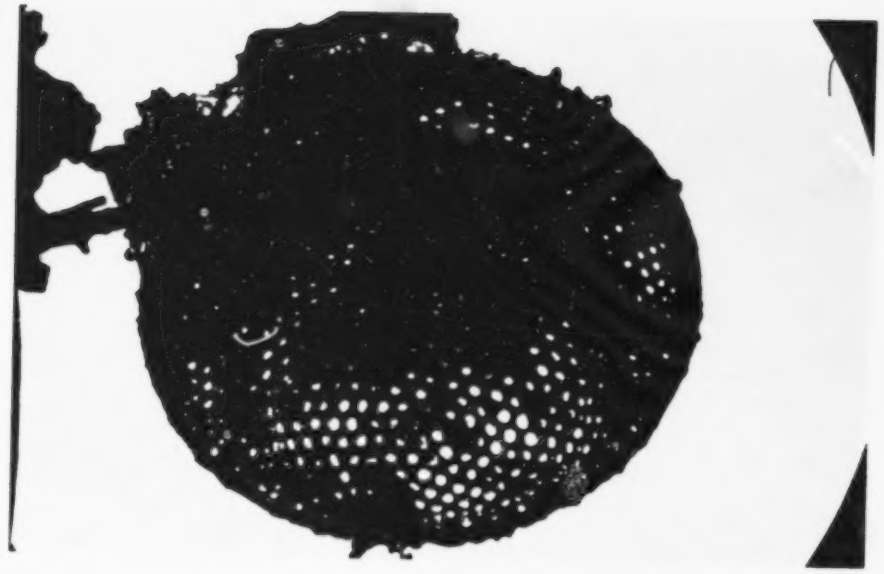

Fig. 13. An electron micrograph of an individual diatom from a typical diatomite, giving an indication of the straining potential of the media (4200×).

Fig. 14. An electron micrograph of a typical diatomite showing irregular diatom fragments (7800×).

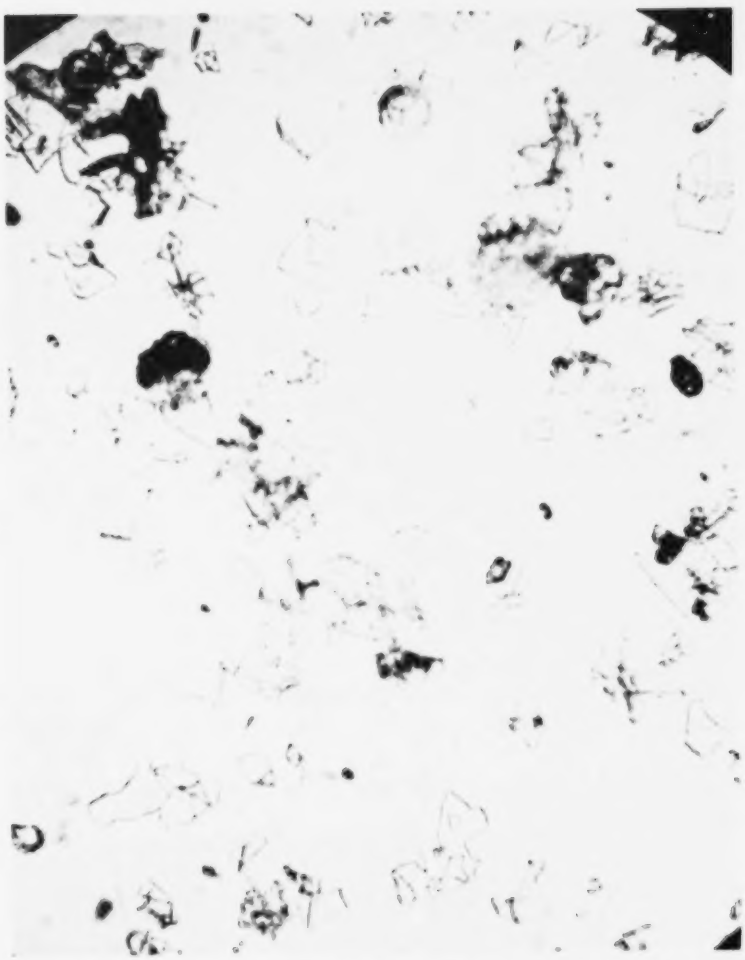

Fig. 15. An electron micrograph of a typical perlite filter aid (400).

APPENDIX I-E

Statistical Analysis[24,25]

Confidence Limits

Tables 9 to 11 list the values of β-indices calculated from the β-prediction equations determined for all the filter aid samples at C_S/C_F levels of 0.02, 0.05 and 0.10. Other statistical calculations outlined in these figures can be understood better with explanation of a few terms:

1. The number of members in any filter aid statistical sample is designated as n.
2. If $X_1, X_2, X_3, \ldots X_n$ are the values associated with the sample size, n, the arithmetic mean of all these values (i.e., $(X_1 + X_2 + X_3 + \ldots + X_n)\ 1/n$) is designated by $\bar{X}$.
3. The sums of squares of differences of these individual values from the mean $\bar{X}$ is designated by the symbol Σx^2. Mathematically, $\Sigma x^2 = (X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \ldots + (X_n - \bar{X})^2$.
4. The term S^2 in these tables represents the variance of the sample which equals $\Sigma x^2/n-1$. The standard deviation, S, is the square root of the variance.
5. The mean of the population is represented by μ to distinguish it from $\bar{X}$, which is the mean of the sample tested. Thus, $\bar{X}$ is the true estimate of μ.
6. The term $S_{\bar{X}}$ equals $\frac{S}{\sqrt{n}}$ and is used in interval estimation of the mean of the population.
7. CI_{95} means the 95% confidence interval of μ.

It has been demonstrated that the term $\bar{X} - \mu/S_{\bar{X}}$ follows a "t" distribution with n - 1 degrees of freedom. This concept can be used to compute confidence intervals on μ. Thus,

$$-t_{(\alpha/2,\ n-1)} < \frac{\bar{X}-\mu}{S_{\bar{X}}} < +t_{(\alpha/2,\ n-1)} \left\{ \begin{array}{l} \text{for} \\ \text{central} \\ \text{confidence} \\ \text{interval} \end{array} \right.$$

As discussed earlier, 95% confidence intervals of μ were calculated, so that α was equal to 5% or 0.05. This concept was used for interval estimation of μ for the respective populations S2, S3, etc. This entire concept, however, is subject to the assumptions of normality of populations and random sampling.

The upper and lower 95% confidence limits for values of μ are entered in Tables 9-11. These confidence limits are represented graphically in Fig. 10. The actual range of the predicted values of β determined in this study is also shown in Fig. 10.

Tests for Equality of Population Means

The purpose of these tests was to determine whether the means of the populations were equal, i.e., if the mean filter cake resistance developed by the various filter aids was statistically equal on the basis of the results of this study. The tests were performed at a 95% confidence level. A prerequisite to the application of these tests is that the variance of different populations should be equal. Thus, equality of variances was also tested for the populations at three designated levels of C_S/C_F. Hartley's "maximum F" test was used for this purpose.

The results of these tests indicated that the variances were statistically equal. Having established the equality of variances, statistical

tests were conducted to see if the means were homogeneous. The F test was used for this purpose. If the two parameters, the mean and the variances of all the populations, are equal, this means that all the filter aids constitute one population and the differences in the values of means are due to chance only. The F test is based on determination of the ratio of variances using both the sample to sample differences and the within the sample differences, since both are independent estimates of the same population. Their ratio is then an F ratio. This is, briefly, the principle of "equal means" test. The calculations done in the following pages illustrate this point. The results of these tests indicated that the available data did not provide any evidence on which it could be concluded that all the means were equal. Thus, the filter aids did not produce equal mean filter cake resistance. This was found true at all three levels of C_S/C_D.

This conclusion led to a new question. Did any of the filter aids develop the same filter cake resistance? In other words, statistically which μ's are equal and which are unequal?

R. A. Fisher's method was used to make a multiple mean comparison. This approach is based on finding the least significant difference between all pairs of means. Means with an actual difference exceeding the corresponding least significant difference, hereafter referred to as LSD, are considered significantly different.

It is convenient, in practice, to arrange the observed sample means in an ordered array and to <u>place means which are not significantly different over the same horizontal line</u>. Means over different horizontal lines are significantly different.

The formula for Fisher's LSD at α level of significance ($\alpha = 0.05$ for 95% confidence level) is:

$$LSD_\alpha = t_\alpha \sqrt{S^2(1/n_1 + 1/n_2)}$$

where

S^2 is error mean square,

t_α is the α level two tail t for error degrees of freedom,

n_1 and n_2 are the sample sizes associated with the two means being compared.

The calculations and the results of these tests are shown in the following pages.

Equal mean tests were also performed on filter aids representing different production lots of filter aid S2, the results of which are also included in this Appendix. For convenience, the common multiple, 10^6, in the values of the filter cake resistance β, is omitted in the analyses to follow.

Calculations for "Equal Mean" Tests Performed at 95% Confidence Level

Tests for equality of variances

When $C_S/C_F = 0.02$

H_o* $(\sigma_1^2 = \sigma_2^2 = ... = \sigma_{11}^2)$

*H_o in statistical language means the hypothesis to be tested. H_a in statistical language means the alternative hypothesis. If H_o is rejected, H_a is accepted. σ^2 denotes the variance of the population to distinguish it from S^2, which is the variance of the sample. Since there are eleven filter aids to be compared, $(\sigma_1^2 = \sigma_2^2 = ... = \sigma_{11}^2)$ is H_o.

H_a (some σ^2's are unequal)

Using Hartley's maximum F test

$F_{maximum}$ = largest S^2/smallest S^2

= 4.84/0.095 = 50.94 (From Table 9)

The average number of degrees of freedom =

Σ degrees of freedom for all the filter aids/

total number of filter aids = 31/11 $\tilde{=}$ 3.

For 11 treatments (number of populations) and 3 degrees of freedom, the region of rejection, Δ, of the hypothesis H_o is

$$\Delta : F_{max} \geq 114$$

Since the value of F_{max} observed is 46.7, which is less than 114, the hypothesis H_o is accepted. Hence, the variances of all populations are statistically equal.

When $C_S/C_F = 0.05$,

H_o ($\sigma_1^2 = \sigma_2^2 = ... = \sigma_{11}^2$)

H_a (some σ^2's are unequal)

F_{max} = 223.9/2.811 = 79.69 (From Table 10)

$$\Delta : F_{max} \geq 114 .$$

Hence, the hypothesis that population variances are equal is again accepted.

When $C_S/C_F = 0.10$

H_o ($\sigma_1^2 = \sigma_2^2 = ... = \sigma_{11}^2$)

H_a (some σ^2's are unequal)

$F_{max} = 3000.49/26.6 = 112.8$ (From Table 11)

$\omega : F_{max} \geq 114$.

Hence, H_o ($\sigma_1^2 = \sigma_2^2 = ... = \sigma_{11}^2$) is again accepted.

Tests for equality of means, assuming normal populations and homogeneous variances

When $C_S/C_F = 0.02$,

H_o ($\mu_1 = \mu_2 = ... = \mu_{11}$)

H_a (all μ's are not equal). (See Table 9.)

Within-sample sum of squares =

$(4.625 - 4.750)^2 + ... + (5.441 - 4.750)^2$

(from within sample 1, i.e., S2)

$+ (7.550 - 7.253)^2 + ... + (6.621 - 7.253)^2 + ...$

(from within sample 2, i.e., S3)

$+ (15.470 - 15.53)^2 + ... + (13.36 - 15.53)^2$

(from within sample 11, i.e., G1)

$= 40.098.$

Total sum of squares about the grand mean,

i.e., mean filter cake resistance of all the filter aids =

$(4.625 - 9.265^*)^2 + ... + (5.441 - 9.265)^2$

$+ (7.550 - 9.265)^2 + ... + (6.621 - 9.265)^2 + ...$

$+ (15.470 - 8.724)^2 + ... + (13.36 - 9.265)^2$

$= 473.968.$

---

*9.265 is the mean filter cake resistance of all the filter aids calculated from the data in Table 9.

Therefore, between-samples sum of squares = 473.968 - 40.098 = 433.870.

Analysis of Variance for the Data of Table 9

Source of variation	D.F.[a]	Sum of squares	Mean square[b]
Between samples	10	433.870	43.387
Within samples	31	40.098	1.293
Total	41	473.968	

$$F \text{ ratio} = \frac{43.387}{1.293} = 33.55$$

$$w = F \geq 2.50$$

[a] D.F. means degrees of freedom.

[b] Mean square = sum of squares/D.F.

Hence, the hypothesis H_o that the means are equal is rejected in favor of the alternative hypothesis that the means are not equal.

When $C_S/C_F = 0.05$,

H_o: $(\mu_1 = \mu_2 = \mu_3 = ... = \mu_{11})$

H_a: (some means are not equal).

With similar calculations, analysis of variance for the data of Table 10 is as below.

Analysis of Variance for the Data of Table 10

Source of Variation	D.F.	Sum of squares	Mean square
Between samples	10	12,721.7	1272.1
Within samples	31	915.3	29.52
Total	41	13,637.0	

$$F \text{ ratio} = \frac{1272.1}{29.52} = 43.09$$

$\omega = F \geq 2.50$

Hence, the hypothesis that the means are equal is rejected in favor of the alternative hypothesis. Again, all the means are not equal.

When $C_S/C_F = 0.10$,

H_o: $(\mu_1 = \mu_2 = \mu_3 = ... = \mu_{11})$

H_a: (some means are not equal).

Analysis of variance for the data of Table 11 is shown below.

Analysis of Variance for the Data of Table 11

Source of variation	D.F.	Sum of squares	Mean square
Between samples	10	169,718.36	16,971.8
Within samples	31	18,194.95	586.93
Total	41	187,913.31	

$$F \text{ ratio} = \frac{16,971.8}{586.93} = 28.91$$

$\omega : F \geq 2.50$

Once again, the hypothesis, H_o, that the means are equal is rejected in favor of the alternative hypothesis.

Multiple Mean Comparisons

The following example illustrates the method for multiple mean comparison.

It is desired to determine whether the mean values of S2 and S3 are equal when $C_S/C_F = 0.02$.

Formula: $LSD_\alpha = t_\alpha \sqrt{s^2(1/n_1 + 1/n_2)}$

Given: $t_\alpha = 2.04$ (for 31 degrees of freedom)

$s^2 = 1.293$ (from analysis of variance, page 100)

$n_1 = 8$

$n_2 = 4$.

Therefore,

$$LSD_\alpha = 2.04 \times \sqrt{1.293(1/8 + 1/4)}$$

$$= 1.41.$$

Least significant difference = 1.41 but the actual difference between the means of S2 and S3 is 7.253 - 4.750 = 2.503, which is greater than 1.41. Therefore, the mean values of S2 and S3 are significantly different. With similar calculations for all pairs of means, the results were calculated at three designated levels of C_S/C_F and are tabulated in Tables 16-18.

Table 16. Ordered array of sample means from Fig. 9 and indications of significant differences among the means when $C_S/C_F = 0.02$.

Filter aid identification	S2	S3	S4	J4	E6	J3	J0	G4	E5	E2	G1
Size of sample n = 8		4	3	3	3	3	3	5	3	4	3
Mean value of $\bar{x}$ in units of 10^6 ft^{-2}	4.75	7.25	7.91	8.17	8.22	8.43	9.68	11.45	12.56	13.34	15.53

Table 17. Ordered array of sample means from Fig. 10 and indications of significant differences among the means when $C_S/C_F = 0.05$.

Filter aid identification	S2	S3	J4	S4	J3	E6	J0	G4	E5	E2	G1
Size of sample n = 8		4	3	3	3	3	3	5	3	4	3
Mean value of $\bar{x}$ in units of 10^6 ft^{-2}	33.17	45.38	48.50	49.99	50.83	57.69	64.20	68.33	76.01	84.31	84.81

Table 18. Ordered array of sample means from Fig. 11 and indications of significant differences among the means when $C_S/C_F = 0.1$.

Filter aid identification	S2	S3	J4	J3	S4	E6	G4	J0	E5	G1	E2
Size of sample n = 8		4	3	3	3	3	5	3	3	3	4
Mean value of $\bar{x}$ in units 10^6 ft^{-2}	143.2	180.8	186.8	196.2	204.1	251.1	263.4	272.6	295.3	307.6	336.4

Calculations for "equal means" tests for different production dots of 52

Test for equality of variances

When $C_S/C_F = 0.02$,

H_o: $(\sigma_1^2 = \sigma_2^2 = \sigma_3^2)$

H_a: (all variances are not equal)

F_{max} = maximum S^2/minimum S^2

= 0.095/0.008 = 11.875 (from Table 19)

Average number of degrees of freedom = 4

Number of samples = 3

$\therefore$ R: $F_{max} \geq 15.5$.

Hence, the hypothesis of equality of variances is accepted.

When $C_S/C_F = 0.05$,

H_o: $(\sigma_1^2 = \sigma_2^2 = \sigma_3^2)$

H_a: (all variances are not equal)

F_{max} = 3.27/.1375 = 23.78 (from Table 20)

R: $F_{max} \geq 15.5$.

Thus, the hypothesis that the variances are equal is rejected at 95% confidence level. But R: $F_{max} \geq 37$ at 99% confidence level. Hence the hypothesis of equal variances is accepted at 99% confidence level. Homogeneity of variances will be assumed for the tests to follow.

When $C_S/C_F = 0.10$,

H_o: $(\sigma_1^2 = \sigma_2^2 = \sigma_3^2)$

H_a: (all variances are not equal)

F_{max}: = 170.43/37.184 = 4.583 (from Table 21)

R: $F_{max} \geq 15.5$.

Table 19. Values of β, predicted at $C_S/C_D = 0.02$, for different production lots of filter aid S2.

	S2 (all bags)	S2 (production lot 1)	S2 (production lot 2)
β values/10^6	4.625	4.687	4.506
	4.469	4.469	4.584
	4.687	4.625	4.890
	4.796		4.796
	4.506		
	4.890		
	4.584		
	5.441		
ΣX	37.998	13.781	18.776
$\bar{X}$	4.750	4.594	4.694
Σx^2	0.665	0.016	0.096
S^2	0.095	0.008	0.032

Table 20. Values of β, predicted at $C_S/C_D = 0.05$, for different production lots of filter aid S2.

	S2 (all bags)	S2 (production lot 1)	S2 (production lot 2)
β values/10^6	34.29	33.95	31.90
	34.69	34.69	29.60
	33.95	34.29	33.83
	33.18		33.18
	31.90		
	33.83		
	29.60		
	33.93		
ΣX	265.37	102.93	128.51
$\bar{X}$	33.171	34.31	32.13
Σx^2	19.678	0.275	9.80
S^2	2.811	0.1375	3.27

Hence, the hypothesis that the variances are equal is accepted.

Tests for equality of means assuming normal populations and homogeneous variances on different production lots of S2

Table 21. Values of β, predicted at $C_S/C_D = 0.1$, for different production lots of filter aid S2.

	S2 (all bags)	S2 (production lot 1)	S2 (production lot 2)
β values/10^6	157.5	149.6	137.1
	161.3	161.3	121.5
	149.6	157.5	145.4
	139.1		139.1
	137.1		
	145.4		
	121.5		
	133.7		
ΣX	1145.2	468.4	543.1
$\bar{X}$	143.15	156.13	135.8
Σx^2	1193.04	74.369	282.07
S^2	170.43	37.184	94.02

When $C_S/C_F = 0.02$,

H_o: $(\mu_1 = \mu_2 = \mu_3)$

H_a: all the means are not equal.

Analysis of Variance of the Data of Table 19

Source of variation	D.F.	Sum of squares	Mean square
Between treatments	2	0.083	0.0415
Within treatments pooled (error)	12	0.770	0.0642
Total about grand mean	14	0.853	

F ratio = 0.0415/0.0642 = 0.646

λ : $F_{max} \geq 5.1$

Hence, the hypothesis that the means are equal is accepted.

When $C_S/C_F = 0.05$,

H_o: $(\mu_1 = \mu_2 = \mu_3)$

H_a: all the means are not equal.

Analysis of Variance of the Data of Table 20

Source of variation	D.F.	Sum of squares	Mean square
Between treatments	2	8.720	4.36
Within treatments Pooled (error)	12	29.753	2.479
Total about grand mean	14	38.473	

F ratio = 4.36/2.479 = 1.759

α : $F_{max} \geq 5.1$.

Hence, the hypothesis that the means are equal is accepted.

When $C_S/C_F = 0.10$,

H_o: $(\mu_1 = \mu_2 = \mu_3)$

H_a: (all the means are not equal).

Analysis of Variance of the Data of Table 21

Source of variation	D.F.	Sum of squares	Mean square
Between treatments	2	741.340	370.670
Within treatments Pooled (error)	12	1549.479	129.125
Total about grand mean	14	2290.819	

F ratio = 370.67/129.125 = 2.871

α : $F_{max} \geq 5.1$

Hence, the hypothesis that the means are equal is accepted.

From the above tests it follows that the mean values of β-indices for all the production lots of S2 considered in this study are equal over the C_S/C_F range studied.

APPENDIX I-F

Table 22. Summary of the values of the exponent "a" for various filter aids.

	S2	S3	S4	J4	J3	J0	E6	E5	E2	G4	G1
Values of "a"	10.385	10.301	10.509	10.213	10.149	10.680	10.502	10.513	10.560	10.363	10.195
	10.450	10.168	10.298	10.391	10.291	10.238	10.476	10.314	10.513	10.489	10.355
	10.331	10.341	10.234	10.047	10.301	10.596	10.595	10.454	10.493	10.329	10.484
	10.241	10.206							10.574	10.280	
	10.274									10.402	
	10.269										
	10.121										
	10.131										
Σ	82.202	41.016	31.041	30.651	30.741	31.514	31.573	31.281	42.140	51.863	31.034
$\bar{X}$	10.275	10.254	10.347	10.217	10.247	10.505	10.524	10.427	10.535	10.372	10.345

Table 23. Summary of the values of the exponent "b" (slope) for various filter aids.

	S2	S3	S4	J4	J3	J0	E6	E5	E2	G4	G1
Values of "b"	2.188	2.044	2.203	1.974	1.906	2.210	2.127	1.967	2.021	1.912	1.772
	2.243	1.939	1.976	2.064	1.990	1.883	2.100	1.929	2.006	2.029	1.831
	2.156	2.065	1.933	1.804	1.971	2.124	2.155	1.990	1.976	1.936	1.974
	2.098	1.941							2.031	1.893	
	2.137									1.992	
	2.107										
	2.037										
	2.005										
Σ	16.971	7.989	6.112	5.842	5.867	6.217	6.382	5.886	8.034	9.762	5.577
$\bar{X}$	2.121	1.997	2.037	1.947	1.956	2.072	2.127	1.962	2.009	1.952	1.859

Table 24. Predicted values of $\beta/10^6$ at indicated levels of C_S/C_F, assuming a common value of "a" (Table 16) for each different type and grade of the filter aid.

S2	S3	S4	J4	J3	J0	E6	E5	E2	G4	G1
When C_S/C_F = 0.02										
4.677	7.228	7.709	8.222	8.453	9.661	8.11	12.27	13.65	11.32	15.38
When C_S/C_F = 0.05										
32.73	45.50	49.77	48.19	50.47	64.42	57.11	75.34	86.10	67.92	84.53
When C_S/C_F = 0.1										
142.6	180.7	204.2	186.2	195.4	271.3	249.5	291.7	335.7	263.0	306.2

APPENDIX I-G

Sample Calculations Involved in Preparing Table 13

Problem: Determine the body feed requirement using S2 filter aid required to give a run length of 6 hours with 100 ft of water terminal headloss when removing 4 mg/l of iron from water at $68^{\circ}F$.

Constants:

(Table 3)

$C_S = 4.0$ mg/l Fe^{+++}

$W = 0.15$ lb/ft^2 (precoat)

Temperature = $68^{\circ}F$, $\nu = 3.898 \times 10^{-2}$ ft^2/hr

$H_c = 100$ ft of water

$t = 6$ hours

$q = 1$ gpm/ft^2 or 8.0214 ft/hr

$g = 32.2$ ft/sec^2 or 417.31×10^6 ft/hr^2

$\gamma_w = 62.4$ lb/ft^3

$\gamma_c = 10.61$ lb/ft^3 (for S2 filter aid).

Case 1. For flat septa

Case 2. For 3½ in. O.D. cylindrical septa

Case 1. Flat Septa.

$$H_c = \sigma \times \qquad \text{(7) Table 3}$$

where

$$\sigma = \frac{q^2 \nu \beta C_F}{g}$$

x = 6 hours, ignoring the initial dilution effect which is negligible

$$H_c = 100$$

$$\therefore\ 100 = \frac{q^2 \nu \beta C_F}{g} \times 6.$$

Since q, g, and ν are constants, it is possible to find βC_F.

$$\beta C_F = \frac{100\ g}{6\ q^2 \nu} = \frac{100 \times 417.31 \times 10^6}{6 \times (8.0214)^2 \times 3.898 \times 10^{-2}}$$

$$\underline{\underline{\beta C_F}} = \underline{\underline{2.775 \times 10^9}}\ ft^{-2}.$$

As long as βC_F <u>using any filter aid</u> is equal to 2.775×10^9 ft^{-2}, the filter operating characteristics will be as specified in the problem. Therefore, using S2 filter aid, it is necessary to find the combination of C_F and β (when $C_S = 4.0$) which will give a βC_F value of 2.775×10^9 ft^{-2}.

Given: $\beta C_F = 2.775 \times 10^9$ and

$$\beta = (10)^a (C_S/C_F)^b$$

$$= 10^{10.275} (C_S/C_F)^{2.121}. \quad \text{(Table 12)}$$

Therefore,

$$\frac{\beta C_F}{C_F} = \frac{2.775 \times 10^9}{C_F} = 10^{10.275}\ \beta\ (\frac{4}{C_F})^{2.121}$$

$$= \frac{2.775 \times 10^9}{C_F} = 1{,}884 \times 10^7 \times \frac{4^{2.121}}{C_F{}^{2.121}}$$

and $$C_F{}^{1.121} = \frac{1{,}884 \times 10^7 \times 18.92}{2.775 \times 10^9}$$

$$C_F{}^{1.121} = 128.5$$

$$C_F = 76\ mg/l \quad \text{(Table 13)}$$

Case 2: 3½ in. O.D. Cylindrical Septa.

$$H_c = \frac{R_s \sigma}{\phi} \text{Ln}\left(1 + \frac{R_S \phi x}{R_o^2}\right) \qquad \text{(5) Table 3}$$

where $R_S = \frac{3.5 \text{ in.}}{2 \times 12 \text{ in./ft}} = 0.1458 \text{ ft},$

$$R_o = R_S + \frac{W}{\gamma_p} = 0.1458 + \frac{0.15}{\gamma_p}$$

and $$\frac{\sigma}{\phi} = \frac{q^2 \nu \beta C_F}{g} \times \frac{\gamma_p}{2q\gamma_w C_F 10^{-6}}$$

$$= \frac{q \nu \gamma_p \beta}{2g\gamma_w (10^{-6})}$$

$$= \frac{8.0214 \times 3.898 \times 10^{-2} \times \gamma_p \beta}{2 \times 417.31 \times 10^6 \times 62.4 \times 10^{-6}}$$

$$= 6.00 \times 10^{-6} \gamma_p \beta$$

and $$\text{Ln}\left(1 + \frac{R_s \phi X}{R_o^2}\right) =$$

$$\text{Ln}\left[1 + (0.1458) \times \left(\frac{2 \times 8.0214 \times 62.4 \times C_F \times 10^{-6}}{\gamma_p}\right) \times (6) \times \frac{1}{\left(0.1458 + \frac{0.15}{\gamma_p}\right)^2}\right]$$

or, $$\text{Ln}\left[1 + (0.1458) \times (2 \times 8.0214 \times 62.4 \times C_F \times 10^{-6}) \times (6) \times \frac{\gamma_p}{(0.1458\ \gamma_p + 0.15)^2}\right]$$

or, $$\text{Ln}\left[1 + \frac{874 \times 10^{-6}\ C_F \gamma_p}{(0.1458\ \gamma_p + 0.15)^2}\right].$$

In this case, therefore

$$H_c = 0.1458 \times 6.00 \times 10^{-6} \gamma_p \beta \text{ Ln}\left[1 + \frac{874 \times 10^{-6} C_F \gamma_p}{(0.1458\ \gamma_p + 0.15)^2}\right]$$

Using cylindrical septa, a trial and error solution is indicated. Choose a C_F and compute H_c. When H_c = 100 ft of water, the value of C_F is correct.

Using S2, where $\gamma_p = 10.61$

Let $C_F = 59$ mg/l

$$\beta = 10^{10.275}(C_S/C_F)^{2.121}$$

$$= 1{,}884 \times 10^7 \left(\frac{4}{59}\right)^{2.121}$$

$$= 1{,}884 \times 10^7 \times 0.0033$$

$$= 62 \times 10^6 \text{ ft}^{-2}.$$

Alternate Calculation of β:

$$\beta = 10^{10.275} \times \left(\frac{4}{59}\right)^{2.121}$$

or $\log \beta = 10.275 + 2.121\ (0.602 - 1.771)$

$$= 10.275 + 2.121\ (-1.169)$$

$$= 10.275 - 2.480 = 7.795$$

and $\beta = 62.4 \times 10^6$

Calculate H_c

$$H_c = 0.1458 \times 6.00 \times 10^{-6} \times 10.61 \times 62.4 \times 10^6 \text{ Ln} \left[1 + \frac{874 \times 10^{-6} \times 59 \times 10.61}{(0.1458 \times 10.61 + 0.15)^2}\right]$$

$$= 579.2 \text{ Ln} \left[1 + \frac{874 \times 10^{-6} \times 59 \times 10.61}{(1.70 \times 1.70)}\right]$$

$$= 579.2 \text{ Ln } [1 + 0.1893] = 579.2 \text{ Ln } 1.1893$$

$$= 579.2 \times 0.1734 = 100 \text{ ft}$$

$\therefore C_F = \underline{59}$ mg/l of S2.